

Building a Digital Twin of Agricultural Production Based on Mathematical Modeling of Sustainability and Development Processes

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Abstract: An approach to building a model of a digital twin of agricultural production is proposed. The modeling is based on the principle of sustainable economic development, harmoniously combining two interrelated processes: sustainability and development. Development requires expanding external relations, markets, and developing specialization. However, in cases of crisis, interruption in the supply of raw materials, closure of sales markets and other unfavorable circumstances, there should always remain a part of the economy, closed on itself, using its own resources and working for its consumers.

The paper considers a structural and mathematical model of sustainable development of the agricultural production process, based on the so-called model of a typical production unit. It is a system of algebraic equalities and inequalities that estimate the power of individual components of the process and are based on balance ratios.

Within the framework of the proposed model, it is possible to consider the sustainable development of the economy of individual agricultural regions, exploring both components of the region's economy and achieving their harmonious interaction. This model also allows us to consider the sustainable development of individual settlements, rural settlements based on local farms and private households, as well as individual agricultural production. In each case, it is necessary to ensure a harmonious combination of stability, self-sufficiency, at least food independence of farms and the possibility of their development.

The proposed models, tuned to a specific online-managed production, can form the basis for the digitalization of agricultural production and serve as a tool to ensure its efficiency and continuous sustainable development.

Keywords: dynamic production models, industry 4.0, forecasting, agricultural production, digital twin, identification, sustainable development

1. INTRODUCTION

The growing problems associated with the unpredictable emergence of global crisis processes and situations, inconsistencies between sectors of the domestic economy and emerging digital transformations in most areas of activity, actualize the search for effective solutions for the strategic management of the digital transformation process in any organization, including large and medium agribusiness companies. It should be noted that in the modern world, sustainable development of agricultural enterprises is based on the use of digital technologies. However, in the digital economy, the elements of the mechanism of the digital transformation process in

agricultural organizations remain not fully defined, only a set of factors that determine the objective need for digitalization are constant, namely: ensuring the country's food security by significantly increasing yields and labor productivity, reducing material costs, protecting environment and so on.

Today, with the transition of the economy to a management system, which is based on the use of a large array of data generated by digital technologies, the issue of revising the targets, principles, mechanisms and tools of modernization in the agricultural industry becomes relevant, since the current practice of digital transformation does not contribute to a technological breakthrough in the studied industry [1]. The rapid and widespread introduction of digitalization, as a natural regularity of the functioning of economic systems in the modern world, presupposes a conceptual and methodological substantiation of the digitalization implementation process, taking into account: the peculiarities of the transition to a digital economy in the agricultural industry and the establishment of the readiness level of large and medium-sized companies in the agribusiness sector management to digital transformation [5,6].

As you know, one of the key areas of digitalization is the use of the so-called digital twin [8], which is based on modeling the production process in real time [2,5]. This paper proposes approaches to the application of mathematical models of agricultural production to build a digital twin of the production process that reflects online the main processes of sustainable development of production and allows you to visually represent the state of the production process and promptly make strategic management decisions.

2. AGRICULTURAL PRODUCTION IN THE MODEL OF SUSTAINABLE DEVELOPMENT OF THE REGION

The problem of sustainable development [7] of regions is associated with the provision of two parallel processes taking place in the region - this is the process of development and the process of ensuring sustainability [9]. Development is closely related to the expansion of production, sales markets, cooperation with external suppliers and buyers. However, these processes lead to risks associated with factors external to the region, and therefore poorly controlled by the region. Stability is determined by the degree of independence of the economy from external factors [10]. Therefore, the less the economy is connected with external suppliers and consumers and the more it is "tied" to the domestic market, the more stable it is. However, in this case, the development of the economy is limited by the volume of the domestic market and stops at some point. Hence, it follows that for the sustainable development of the region's economy, it is necessary to harmoniously combine both of these processes. For its development, the region must expand external relations, look for its niche in foreign markets, and specialize in them. However, in the event of a crisis, cessation of supplies from outside of raw materials, components, closure of sales markets and other unfavorable circumstances, a part of the economy should remain in the region, closed on itself, using its own resources and working for its consumers within the region. Only in this case can we talk about sustainable development of the region.

The sustainable development of a region is closely linked to its economic security. As you know, food security is one of the main directions of ensuring the national security of the country. Therefore, agricultural production plays an important role in the sustainable development of the region. At the same time, the sustainable development of the agricultural sector can be considered as an independent problem within the framework of the economic security of the region. Agricultural production naturally contains both components of the sustainable development process. First of all, this is the production of products for domestic consumption - the consumption of agricultural products by the population of the region itself - this process makes a certain contribution to the economic stability of the region. And besides this, the agricultural industry produces products for the external consumer - these are grain,

dairy products, and other types of products supplied to external consumers. And the development of the industry in the direction of markets external to the region, its specialization, works to enhance the economic power of the region as a whole.

Within the framework of the sustainable development model proposed in this work, it should be noted that the agricultural industry has a sufficiently developed part of ensuring sustainability, since for the most part it uses its own resources - land, water bodies and others and works for internal consumption, which gives a certain independence from extra-regional factors. At the same time, there is also a component associated with external markets and allowing active development.

3. MATHEMATICAL MODEL OF AGRICULTURAL PRODUCTION AS A TYPICAL PRODUCTION FACILITY

The modeling of agricultural production is based on a methodology that uses the concepts of the capacity of complex systems, the dynamics of an economic object, a typical production facility (TPF), as a structural and mathematical model of business processes [11-13,15,16]. When talking about a process, then, as a rule, there is an input through which something enters the process, and, after processing, the result of the process is obtained at the output. However, in addition to this, fixed assets are involved in the process, i.e. the equipment with which the process is carried out, as well as the personnel involved in the implementation of the process [18]. An important role is played by the information and intellectual component, which, in the case of the production process, is technology, technical regulations, licenses, intangible assets, etc. The structural part of the TPF model (Fig. 1) is close to the models of business processes of the SADT methodology, including the IDEF0 standard [14]. However, the TPF, in addition to the structural component, also contains a mathematical model.

Further, we will consider the TPF in relation to agricultural production, which we will call an agricultural production facility (APF). Agricultural production facilities can include both individual agricultural production, farms, and agriculture of a district or region, and even an agro-industrial complex as a whole.



Fig. 1. Typical production facility.

Let some agricultural production facility produces n types of products, including intermediate products, the volumes of which for a given time interval will be denoted by the column vector $X = (x_1, \dots, x_n)^T$. Some of these products are used for domestic consumption. To describe the processes of consumption, we will apply the model of the inter-industrial balance of V. Leontiev (IIB) [3]. Let us introduce a square $n \times n$ - matrix A_x , which determines the norms of expenditures on domestic consumption, which is called the matrix of coefficients of direct production costs.

Let us introduce an m -dimensional column vector $Y = (y_1, \dots, y_m)$, the components of which represent the volumes of products or services provided by the APF to external consumers. Here, in contrast to the IIB model, the nomenclature of the external product Y and internal X are different, but the production of the external product is based on the internal product, and the amount of the internal product required to produce the output product is equal

to $A_y Y$, where A_y is the $n \times m$ matrix of consumption rates domestic product per unit of output product. In the special case when the vectors X and Y coincide, A_y is the identity matrix. The volume of domestic production X is related to the volume of output Y by an expression similar to the IIB equation:

$$(E - A_x)X = A_y Y, \tag{3.1}$$

where E is the n -dimensional identity matrix. Expression (3.1) within the framework of the TPF structure is a mathematical model of the block of the APF production process.

Consider the rest of the TPF blocks (Fig. 1).

The "Output" block in the TPF structure characterizes the result of the process and determines the requirements for all other blocks of the production process. The output determines the characteristics of the products. It contains both actual information about the release and the required characteristics of the manufactured product. These are the production volumes, market prices and lead times, etc. They entail requirements for the rest of the units of the production unit. Then, having set some required values of the output $Y = Y_{rv}$, it is possible, on the basis of expression (3.1), to determine the requirements for the volume of output of the domestic product X_{rv} .

Block "Entrance" is the process of organizing the supply of raw materials, accessories, components coming from outside. This includes, for example, the purchasing processes that take place in the APF. The stability of this block significantly affects the stability of the whole APF.

Let's introduce the r -dimensional vector column $V = (v_1, \dots, v_r)$ of the volumes of products coming from outside. The need for these products is related to the vector X using the $r \times n$ -matrix of the cost rates A_v :

$$V = A_v X \tag{3.2}$$

The components of the input vector, as a rule, have restrictions and are described by a system of inequalities of the form:

$$0 < V < V^*, \tag{3.3}$$

where V^* is the vector of the maximum possible volumes of input products for the considered time interval. An expression of the form (3) defines a certain set of possible volumes of input products, which in what follows we will call the input power of the system.

The block "Fixed assets" characterizes the production capacity of ATF, which are determined by the availability of this or that equipment. Let the enterprise have q types of equipment. The productivity of equipment and their number directly determine the possibilities of producing one or another component of the vector X . Let us introduce into consideration the vector $K = (K_1, \dots, K_q)^T$, the components of which correspond to the number of units of operating equipment of each type. It is assumed that each type of equipment can produce several types of products. The capabilities of each piece of equipment, i.e. the power of a piece of equipment is described by the following inequality:

$$0 < a_{i1} x_1 + \dots + a_{in} x_n < 1, i = 1, \dots, q, \tag{3.4}$$

Here the coefficients a_{ij} are related to the productivity of the i – th equipment for the j – th type of product. Then the capacity of the equipment, as the set of all possible options for the production of products X , is determined by the following system of equalities and inequalities:

$$\begin{aligned} 0 < a_{i1} x_{1i} + \dots + a_{in} x_{ni} < K_i, i = 1, \dots, q, \\ x_j = x_{j1} + \dots + x_{jq}, j = 1, \dots, n, \end{aligned} \tag{3.5}$$

where the first index of the variable x_{ji} corresponds to the number of the product component, and the second index to the number of the type of equipment on which it is produced.

The "Staff" block contains mathematical models of the capacity of the ATF personnel. Staff power, i.e. capacity of ATF in personnel, represents a set of all the possibilities of production, limited by the presence and number of personnel of one or another qualification. Let l be the number of types of qualifications of personnel involved in the production process. Let us introduce into consideration the vector $L = (L_1, \dots, L_l)^T$, the components of which correspond to the number of employees of a particular qualification. Then, introducing into consideration the coefficients p_{ij} , which characterize the productivity of a specialist of the i – th qualification for the production of the j – $type$ of product, by analogy with the capacity of the equipment, we obtain the expression for the capacity of the personnel of the ATF:

$$\begin{aligned} 0 < p_{i1} x_{1i} + \dots + p_{in} x_{ni} < L_i, i = 1, \dots, l, \\ x_j = x_{j1} + \dots + x_{jq}, j = 1, \dots, n, \end{aligned} \quad (3.6)$$

where the first index of the variable x_{ji} corresponds to the number of the product component, and the second index to the number of the type of personnel qualifications.

The block "Information-Technologies" provides mutual coordination of all processes of the enterprise on the basis of main production technologies, methods and management structure. This block determines the production technology, the availability of documentation and rights to one or another type of activity, permission to perform certain types of work, etc. Therefore, it determines the structure of the matrices, the values of the coefficients included in the formulas considered above. The influence of this block on production activity is carried out through productivity, determined by the availability of a particular technology, the perfection of the management system, as well as the presence or absence of patents and licenses for certain technologies and activities.

Thus, expressions (3.1) - (3.6) represent a mathematical model of the open source production process.

In the proposed mathematical model, the types of available equipment, types of qualifications of specialists, the coefficients of their capacities, as well as the corresponding matrix of cost coefficients A_y, A_v, A_x , should be specified. By specifying the vectors of the number of equipment K and personnel L , as well as the constraints on the input V^* , we obtain the system power Ω_y , which is, ultimately, the set of all possible output vectors Y in the presence of appropriate equipment, personnel, and external supply capabilities. By setting a certain vector $Y = Y_{rv}$, corresponding to the needs of the external market, it is possible to determine whether there is enough capacity to produce the volume of output Y_{rv} in the presence of equipment K and personnel L and constraints on external resources V^* . If the capacity is not enough, then the model allows you to determine the required number of equipment and personnel that must be added to ensure the production of Y_{rv} .

4. AN EXAMPLE OF BUILDING A SUSTAINABLE DEVELOPMENT MODEL FOR AN AGRICULTURAL PRODUCTION SYSTEM

Consider, as an illustrative example, a certain livestock farm that breeds cattle and produces meat and milk.

Let's say there are K_{cow} on a farm.

Let us denote the volume of annual production of meat X_{meat} and milk X_{milk}

X_C calves are born on the farm per year, of which a part in the amount of ΔK_{cow} is used to repair the herd, the rest is either sold or fattened in the amount of X_B for meat production.

Then:

$$X_B < X_C - \Delta K_{cow} \quad (4.1)$$

In the following, for simplicity, we will use the average data:

1) the average annual milk yield per cow is 3.5 tons, i.e. annual milk production, in tons per year, is given by:

$$X_{milk} = 3,5K_{cow},$$

2) a cow gives birth to one calf per year, i.e. $X_C = K_{cow}$,

3) the replacement number of calves, taking into account the case, is determined as 20% of the herd size, i.e.

$$X_C^r = 0,2K_{cow} = 0,2X_C,$$

4) annually, 20% of cows that are used for meat production also drop out of the herd. From one mature cow, 300 kg of meat comes out, and in total, due to the replacement, meat is produced in the amount of:

$$X_{meat}^r = 0,06K_{cow}, \text{ tons per year.}$$

5) the yield of meat from a fattened bull is 200 kg, then the annual production of meat is determined by the expression:

$$X_{meat}^B = 0,2X_B, \text{ tons per year.}$$

In total, the annual production of meat (in tons) is determined by the expression:

$$X_{meat} = X_{meat}^r + X_{meat}^B = 0,06K_{cow} + 0,2X_B \quad (4.2)$$

Consider labor costs. Here we will not describe in detail the structure of labor costs, but only assume that there are conditionally variable and fixed costs. Conditionally variable costs proportional to the number of cows, calves, bulls on the farm. Conditionally fixed costs do not depend on the composition of the herd, are associated with work that does not depend, within certain limits, on the number of cattle on the farm. For example, it is necessary to support a tractor driver, a mechanic, a veterinarian, a watchman, etc. Labor costs, as the total number of staff units required on the farm, are determined by the expression:

$$L = T_{cow}K_{cow} + T_C X_C + T_B X_B + L_0, \quad (4.3)$$

where we assume $T_{cow} = 0,07$, $T_C = 0,05$, $T_B = 0,05$ people (staff units) per head of cattle, $L_0 = 10$ people, which corresponds to the labor costs of some small average farm.

In the future, it is assumed that part of the wages of workers is given out with meat and milk produced on the same farm, and in an amount determined by the consumption norms for all members of the employee's family. Every worker has a family. The average number of people in a family is k_f and for Russia now it is 3-4 people, let's assume $k_f = 4$.

Then the number of people who have to eat the farm products is $k_f L$. The rest that is produced on the farm is an external product that goes on sale. In accordance with the norms, each person consumes $a_{meat} = 0,075$ tons of meat (75 kg) and $a_{milk} = 0,34$ tons of milk (340 kg) per year. Then the annual consumption by the population of the farm of its own products is determined by the expressions:

$$X_{meat}^L = a_{meat} L \text{ tons meat per year}$$

and

$$X_{milk}^L = a_{milk} L \text{ tons milk per year.}$$

Hence, the system of equations describing the production of milk and meat on the farm has the following form:

$$[3,5 - a_{milk}(T_{cow} + T_C)]K_{cow} - a_{milk}T_B X_B = a_{milk}L_0 + Y_{milk},$$

$$[0,06 - a_{meat}(T_{cow} + T_C)]K_{cow} - (0,2 - a_{meat}T_B)X_B = a_{meat}L_0 + Y_{meat} \quad (4.4)$$

where Y_{milk} is the annual milk production program,
 Y_{meat} - annual meat production program.

This model allows solving both direct and inverse problems. The direct problem includes determining the amount of meat and milk produced by the farm for a given number of dairy cows and fattening bulls. Some calculation results are shown in Table 1. For example, a herd of 100 cows, with the maximum number of fattening bulls, i.e. with $K_{cow}=100$ heads and $X_B=80$ heads, yields 20.0 tons of meat and 341.2 tons of milk per year.

Note that in the first line of Table 1, with the number of dairy cows $K_{cow} = 3$ and the number of bulls $X_B = 3$, the meat yield $Y_{meat} = 0$. This means that such a number of animals makes it possible to provide meat only for farm workers, and meat for sale does not remain, although milk is produced for sale.

Table 1. Calculation results

№	K_{cow} , heads	X_B , heads	Y_{milk} tons/year	Y_{meat} tons/year
1	3	3	6,9	0
2	4	3	10,4	0,04
3	10	8	31	1,3
4	20	16	65,5	3,4
5	30	24	100	5,5
6	50	40	169	9,7
7	100	80	341	20
8	200	160	686	41
9	400	320	1375	62

The inverse problem is to determine the required herd size, the number of dairy cows and fattening bulls that would provide the required amount of milk Y_{milk} and meat Y_{meat} put on the market. Some results of the calculations are shown in Table 2. Since the results of the calculations are fractional, herd sizes are shown in brackets, rounded to whole numbers.

Table 2. Calculation results

№	Y_{milk} tons/year	Y_{meat} tons/year	K_{cow} , heads	X_B , heads.
1	10	0	3,85 (4)	2,8 (3)
2	50	0	15,4 (4)	0
3	100	5	30	21,5 (22)
4	400	20	117	75,3 (75)
5	400	25	117	100,7 (101)
6	400	40	117,5 (118)	177,1 (177)

Figures 2 and 3 show the results of calculations of the required number of dairy cows K_{cow} and bulls for fattening X_B (ordinate axis), to ensure a different amount of milk yield Y_{milk} (abscissa axis), provided that meat is not produced for sale, $Y_{meat} = 0$. Figure 2 considers the option when the farm has $L_0 = 10$ permanent workers. Figure 3 considers the option when the farm has $L_0 = 20$ permanent workers.

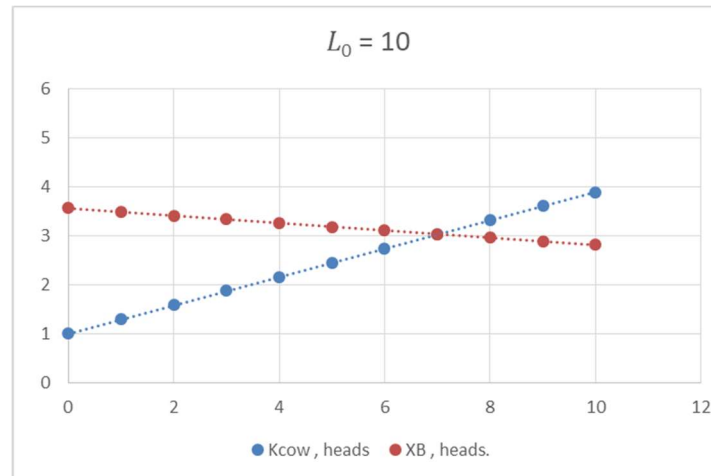


Fig. 2. Calculation results: $L_0 = 10$ permanent workers.

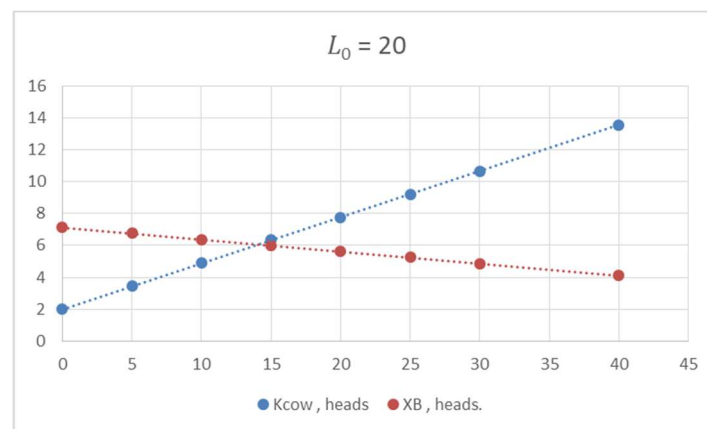


Fig. 3. Calculation results: $L_0 = 20$ permanent workers

The number of cows is shown in blue, and the number of bulls is shown in red. It can be seen from the graphs that, despite the fact that the sale of meat is not provided, it must be produced to provide for the population of the farm. At the same time, in the first case, when $L_0 = 10$ people, if less than 7 tons of milk is produced per year, then the number of bulls is required more than there are cows. And in the second case, when $L_0 = 20$ people, the number of bulls is required more than there are cows, with the demand for milk already less than 13 tons per year.

Thus, in this calculation example, expression (4.4) is a static mathematical model of the production process of a meat and dairy farm, when part of the output is distributed among the workers themselves. It does not specifically consider the financial side of the farm, which is associated with market prices for products and raw materials. It illustrates the simulation of a closed part of the production process, when part of the production is used for internal needs. Then, in the absence of demand for products, the farm continues to work, providing for the needs of its own workers and, thereby, ensuring its sustainability.

As you know, most of the costs in meat and dairy production is animal food. Therefore, this model can also include the production of animal food, and, accordingly, the payment of the labor of animal food production with products produced on the farm. Thus, we will further expand and strengthen the sustainable component of the farm as a production system.

5. CONCLUSIONS

This model allows you to simulate the gradual transformation of agricultural production facility in the following directions.

First, as shown above, the development and expansion of production is modeled, the structure of production is determined, which is necessary to ensure the production of new volumes, or new types of products. At the same time, this can be done both at the expense of previous technologies, increasing the amount of equipment of an existing type, or it is possible to acquire and launch new types of equipment, new technologies, which will affect the size and values of the corresponding matrices and model coefficients.

Secondly, it is possible to simulate the localization of production, which consists in reducing the dimension of the vector V of production received from outside, and expanding the vector X of domestic production. This again leads to the acquisition of new equipment, the adoption of new personnel, i.e. changing the structure and dimensions of the model. In fact, this adds stability to production and creates new jobs.

Thirdly, by changing the matrix A_y , one can vary the ratio of how much production to keep for domestic consumption and how much to send to the external market. Thus, to manage the degree of stability of the enterprise and its development.

In general, within the framework of the proposed model of sustainable development, one can consider not only agriculture, but the entire agro-industrial complex of the region as a whole. On the other hand, within the framework of this model, one can also consider the sustainable development of individual regions, examining both components of the region's economy and achieving their harmonious interaction. The proposed model also makes it possible to consider the sustainable development of individual settlements, for example, rural settlements relying on local farms and private households, as well as individual agricultural production. In each case, it is necessary to ensure a harmonious combination of stability, self-sufficiency, at least food independence of farms and the possibility of their development.

REFERENCES

1. Babkin A.V. et al. (2018). *The methodology for the development of the economy, industry and services in terms of digitization*. Saint Petersburg, Russia: Polytech-press.
2. Bakhtadze, N., Elpashev D., Suleykin, A. & Pyatetsky V. (2021). Digital Ecosystem Situational Control Based on a Predictive Model. *IFAC-PapersOnLine*, **54**(1), 300–306.
3. Eidelman, M.R. (1966). *Interindustry balance of the social product (Theory and practice of its compilation)*. Moscow, Russia: Statistics.
4. Ershov, K.V., Kusharev, A.A. & Sirazetdinov, R.T. (2005). A virtual enterprise for the implementation of repair technologies on technological equipment. *Izvestiya Vysshikh Uchebnykh Zavedenij. Aviatcionnaya Tekhnika*, **4**, 54–56.
5. Hess, T., Matt, C., Benlian, A. & Wiesbock, F. (2016). Options for Formulating a Digital Transformation Strategy. *MIS Quarterly Executive*, **15**(2), 123–139.
6. Kitova, O.V & Bruskin, S.N. (2018). Cifrovaya transformaciya biznesa [Digital transformation of business]. *Cifrovaya ekonomika*, **1**, 20–25, [In Russian].
7. Koptyug, V.A., Matrosov, V.M., Levashov, V.K. & Demyanko, Y.G. (2001). Approaches to the development of a national strategy for sustainable development of Russia. *Chemistry for sustainable development*, **9**, 493–502.
8. Kurganova, N.V., Filin, M.A., Chernyaev, D.S., Shaklein, A.G. & Namiot D.E. (2019). Implementation of digital twins as one of the key areas of digitalization of production. *International Journal of Open Information Technologies*, **7**(5), 105–114.
9. Sirazetdinov, R.T., (2017). Modeling of sustainable enterprise development. *In the collection: Analytical mechanics, stability and control. Proceedings of the XI International Chetaev Conference*, 192–200.
10. Sirazetdinov, R.T., Samodurov, A.V., Khusnutdinov, A.N. & Tarchinskaya, E.N. (2016). Sustainable Development of Manufacturing Enterprises Based on Structural,

Infological and Dynamic Modeling. *10th International Conference on Application of Information and Communication Technologies*, Baku, Azerbaijan, 1–3, <https://doi.org/10.1109/ICAICT.2016.7991783>.

11. Sirazetdinov, R.T. (1998). Mathematical simulation of the capacity of the infrastructure of complex systems. *Journal of Computer and Systems Sciences International*, **37**(3), 438–445.

12. Sirazetdinov, R.T. & Brazhkina, A.A. (2010). Universal structural model of a typical economic cluster. *Management of large systems*, **29**, 152–166.

13. Sirazetdinov, R.T., Samodurov, A.V., Yenikeev, I.A. & Markov, D.S. (2016). Dynamic model of production enterprises based on accounting registers and its identification. *IOP Conference Series Materials Science and Engineering*, **134**(1), 012026

14. Sirazetdinov, R.T. & Rizaev, Z.I. (2012). Algorithms for assessing the feasibility of the project and the distribution of tasks based on mathematical modeling of employee competencies. *Non-linear world*. **10**(5), 317–321.

15. Sirazetdinov, T.K. (1972). A dynamic forecasting model and optimal control of an economic object. *Izvestiya Vysshikh Uchebnykh Zavedenij. Aviatsionnaya Tekhnika*, **4**, 32–38.

16. Sirazetdinov, T.K. (1996). *Dinamicheskoe modelirovanie ekonomicheskikh ob'ektov* [Dynamic modeling of economic objects]. Kazan, Russia: Feng Publishing House, [In Russian].

17. Sirazetdinov, T.K., Rodionov, V.V., Sirazetdinov, R.T. (2005). *Dinamicheskoe modelirovanie ekonomiki regiona* [Dynamic modeling of the region's economy]. Kazan, Russia: Feng Publishing House, [In Russian].

18. Smirnova, G.S., Sabitov, R.A., Korobkova, E.A., Sabitov, Sh.R. (2017). Modeling production facility as a dynamic integrated interacting objects system. *Procedia Computer Science*, **112**, 965–970