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On the Stability of Periodic Difference Inclusions

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Abstract: The paper considers asymptotically stable periodic difference inclusions. The uniform character of the convergence of solutions to zero is established. For selector-linear difference inclusions the equivalence of the uniform asymptotic stability and the uniform exponential stability is proved, and a necessary and sufficient condition for the uniform asymptotic stability in the form of a certain limit relation is obtained. Examples of systems leading to periodic difference inclusions are given. These results can find applications in the stability analysis of control systems with periodic parameters, in particular, servomechanisms whose elements operate on alternating current, control systems with amplitude-frequency modulation, systems used to solve problems associated with the study of large electric power systems in the presence of forced oscillations.

Keywords: periodic difference inclusion, periodic selector-linear difference inclusion, uniform asymptotic stability, uniform exponential stability

1. INTRODUCTION

The problem of the stability of difference and discrete inclusions arises in various areas of mathematics: in control theory, in linear algebra, in the study of convergence of iterative processes, in problems related to discrete wavelet transforms and Markov chains. In some cases, selector-linear inclusions can be used. For example, this is the problem of absolute stability, the study of linear non-stationary systems, the matrix of the right side of which satisfies interval constraints, the study of the stability of control systems that contain elements with incomplete information. Nonlinear discrete control systems were considered in [7,5]. Their equivalence to autonomous selector-linear difference inclusions is proved. Various stability conditions were obtained using the method of Lyapunov functions. For autonomous selector-linear difference inclusions, the Lyapunov indicator was introduced and some properties of solutions were established in [1], asymptotic stability conditions in the form of constraints on the right-hand side of inclusions in [2] and those in the form of the existence of smooth and finite-step Lyapunov functions in [3,4] were obtained.

The problems of absolute and robust stability of discrete control systems with periodically varying parameters were solved in [6,8]. In particular, it was established that the considered control systems with periodic parameters are equivalent, in the sense of the coincidence of the sets of solutions, to time-periodic selector-linear difference inclusions.

In this paper, periodic difference inclusions are considered. It is known that the properties of the uniform asymptotic stability and the uniform exponential stability of zero solution of autonomous selector-linear difference inclusions are equivalent. For periodic selector-linear difference inclusions, the question of the equivalence of these properties has not previously been considered.

The remainder of this paper is structured as follows. In Section 2, we consider periodic difference inclusions of general form and periodic selector-linear difference inclusions. We

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give preliminary remarks and definitions. In Section 3, it is proved that under some restriction of initial conditions, the convergence of solutions of a periodic difference inclusion to zero has the uniform character. We also establish the equivalence of the uniform asymptotic stability and the uniform exponential stability of the zero solution for periodic selector-linear difference inclusions. Using the variational method for periodic selector-linear difference inclusions, a necessary and sufficient condition for uniform asymptotic stability is obtained in the form of a limit relation. In Section 4, two examples of systems leading to periodic difference inclusions are given. In Section 5, we offer concluding remarks.

2. STATEMENT OF THE PROBLEM

Consider the dynamic systems described by periodic difference inclusion (2.1)

$$x(s+1) \in F(s,x), \quad F(s,x) = F(s+M,x), \tag{2.1}$$

where $s = 0,1,...,$ $x \in R^n$, $M \in N$, N is the set of natural numbers. Everywhere below we assume that in some domain

$$G = \{0 \le s \le M,\ x \in G_R,\ G_R = \{x_0 : \|x_0\| \le R\}$$

for all $(s,x) \in G$ the set $F(s,x) \subset R^n$ is nonempty, bounded, closed, convex, and the multivalued function $F : R^{n+1} \to R^n$ is upper semicontinuous. We will call a solution $x(s)$ of inclusion (2.1) a sequence of vectors $\{x(s)\}$, satisfying for all $s \in N$ inclusion (2.1). We assume that the sequence $\{x(s)\}: x(s) = 0,\ s \in N$ is the equilibrium position $x(s) \equiv 0$ of the inclusion (2.1). Due to the multivaluedness of the function $F(s,x)$ a point (s_0, x_0), generally speaking, specifies not one but a set of solutions. Due to the periodicity of the multivalued function $F(s,x)$ in s when studying the properties of solutions $x(s,s_0,x_0)$ of inclusion (2.1), without loss of generality, we can assume that $0 \le s_0 \le M$.

Definition 2.1:
Inclusion (2.1) is stable if for any $\varepsilon > 0$ *there exists* $\delta(x(s,s_0,x_0),s_0,\varepsilon) > 0$ *such that, as soon as the initial conditions* $x(s_0) = x_0$ *satisfy the condition* $\|x_0\| < \delta(x(s,s_0,x_0),s_0,\varepsilon)$, *the solution* $x(s,s_0,x_0)$ *with initial condition* x_0 *satisfies the inequality* $\|x(s,s_0,x_0)\| < \varepsilon$ *for all* $s \ge s_0 \ge 0$.

Definition 2.2:
Inclusion (2.1) is asymptotically stable if the conditions of Definition 2.1 are satisfied and the limit relation $\lim_{s\to+\infty} x(s,s_0,x_0) = 0$ *holds true, that is, for any* $\eta > 0$ *there exists* $S(x(s,s_0,x_0),s_0,x_0,\eta) \in N$ *such that for all* $s \ge s_0 + S(x(s,s_0,x_0),s_0,x_0,\eta)$ *the solution* $x(s,s_0,x_0)$ *of inclusion (2.1) satisfies the inequality* $\|x(s,s_0,x_0)\| < \eta$.

Definition 2.3:
Inclusion (2.1) is uniformly asymptotically stable if the conditions of Definitions 2.1 and 2.2 are satisfied and numbers δ *and* S *do not depend on solution* $x(s,s_0,x_0)$, $s_0 \ge 0$ *and* $x_0 \in R^n$.

Consider a periodic selector-linear difference inclusion

$$x(s+1) \in F(s,x), \quad F(s,x) = \{y : y = B(s)x,\ B(s) \in \Omega(s)\},$$
$$\Omega(s+M) = \Omega(s)\,, \tag{2.2}$$

where $s = 0,1,\ldots,\ M \in N,\ x(s) \equiv 0$ is the equilibrium position of the difference inclusion (2.2), $\Omega(s)$ is a convex compact set of real $(n \times n)$ - matrices.

Let $x_B(s,s_0,x_0)$ be a solution of inclusion (2.2) defined by a matrix $B(s) \in \Omega(s)$.

Definition 2.4:

Inclusion (2.2) is uniformly exponentially stable if there exist numbers $\alpha > 0$ and $\beta \geq 1$ such that, for any solution $x_B(s,s_0,x_0)$ of inclusion (2.2), the inequality

$$\|x_B(s,s_0,x_0)\| \leq \beta \|x_0\| \exp(-\alpha(s-s_0))$$

holds true for any matrix $B(s) \in \Omega(s)$, any $s \geq s_0 \geq 0$ and $x_0 \in R^n$.

The problem is to establish for inclusion (2.1) uniform character of the limit relation in Definition 2.2. In the case of inclusion (2.2), the problem is to determine a condition for uniform asymptotic stability and to prove the equivalence of the properties of uniform asymptotic stability and uniform exponential stability.

3. RESULTS

Theorem 3.1:

If inclusion (2.1) is asymptotically stable, then there exists $\delta_0 > 0$ such that all solutions $x(s,s_0,x_0)$ of inclusion (2.1) satisfy the condition $\lim\limits_{s \to +\infty} x(s,s_0,x_0) = 0$ uniformly with respect to (s_0,x_0) for any $0 \leq s_0 \leq M$ and $x_0 : \|x_0\| \leq \delta_0$.

The proof of Theorem 3.1 is carried out mutatis mutandis by the scheme of Theorem 1 in [9].

Let $x_B(s,s_0,x_0)$ be a solution of inclusion (2.2) with the initial conditions (s_0,x_0), corresponding to a matrix $B(s) \in \Omega(s)$. To obtain a condition for uniform asymptotic stability of inclusion (2.2), we introduce into consideration the functions

$$W(t,s_0,x_0) = \max_{B(s) \in \Omega(s)} \|x_B(s,s_0,x_0)\|^2$$

and

$$\rho(s,s_0) = \max_{\|x_0\|=1} W(s,s_0,x_0), \quad s \geq s_0 \geq 0. \tag{3.1}$$

By the Weierstrass theorem, taking into account the form of inclusion (2.2), the function $\rho(s,s_0)$ is defined for any x_0 and $s \in N$ since the functional $\|x_B(s,s_0,x_0)\|^2$ is a continuous function of the variables x_0 and $B(s)$.

Using the function $\rho(s,s_0)$ we can formulate the following criterion for asymptotic stability.

Theorem 3.2:

Inclusion (2.2) is uniformly asymptotically stable if and only if the limit relation

$$\lim_{s\to+\infty} \rho(s,s_0)=0 \tag{3.2}$$

holds true uniformly with respect to $s_0 \geq 0$.

Proof. Necessity. Since inclusion (2.2) is uniformly asymptotically stable, then for any $\eta>0$ there exists $S(\eta)\in N$, such that $\|x_B(s,s_0,x_0)\|<\eta$ for all $s_0\geq 0,\ s\geq s_0+S,$ $B(s)\in\Omega(s),\ x_0:\|x_0\|=1$. From the last inequality and (3.1) follows that $\rho(s,s_0)<\eta^2$ for all $s\geq s_0+S$ and $s_0\geq 0$, which proves the necessity of condition (3.2).

Sufficiency. Due to the uniform boundedness of the elements of the matrix $B(s)$ the solution $x_B(s,s_0,x_0)$ of inclusion (2.2) with $\|x_0\|=1$ will be uniformly bounded for all $s=s_0,s_0+1,...,s_0+s^*$. Therefore, for any $s^*>0$ there exists $\gamma(s^*)\geq 1$, such that $\rho(s,s_0)\leq\gamma(s^*)$ for all $s=s_0,s_0+1,...,s_0+s^*$. It follows from the last inequality and relation (3.2) that there exists, $\rho_0\geq 1$ that is independent of s_0 such that $\rho(s,s_0)<\rho_0$ for all $s\geq s_0\geq 0$. For any $s_0\geq 0$, $x_0:\|x_0\|=1$, $s\geq s_0$, $B(s)\in\Omega(s)$

$$\|x_B(s,s_0,x_0)\|^2\leq\rho(s,s_0)<\rho_0. \tag{3.3}$$

From (3.3), linearity and periodicity of inclusion (2.2) for any $\varepsilon>0$, any $s_0\geq 0$, $x_0:\|x_0\|=1,\ s\geq s_0$ and $B(s)\in\Omega(s)$ follows

$$\left\|x_B(s,s_0,\varepsilon/\sqrt{\rho_0}x_0)\right\|<\varepsilon.$$

We set $\delta(\varepsilon)=\varepsilon/\sqrt{\rho_0}$. Then it follows from (3.3) that solutions of inclusion (2.2) satisfy the inequality $\|x_B(s,s_0,x_0)\|<\varepsilon$ for all $s\geq s_0\geq 0$ and $B(s)\in\Omega(s)$, if only $\|x_0\|<\delta(\varepsilon)$. It follows from (3.2) that for any $\eta>0$ and $R>0$, there exists such $S(\eta,R)\in N$, that is independent of $B(s)\in\Omega(s)$ and $s_0\geq 0$, such that for all

$$\{s_0\geq 0,\ s\geq s_0+S(\eta,R),\ \|x_0\|=1,\ B(s)\in\Omega(s)\}$$

the inequality holds true

$$\|x_B(s,s_0,x_0)\|^2\leq\rho(s,s_0)<\eta^2/R^2,$$

and hence the inequality $\|x_B(s,s_0,Rx_0)\|<\eta$ due to the linearity and periodicity of inclusion (2.2). Therefore, $\|x_B(s,s_0,x_0)\|<\eta$ for all

$$\{s_0\geq 0,\ s\geq s_0+S(\eta,R),\ \|x_0\|=1,\ B(s)\in\Omega(s)\}.$$

Thus, the solution of inclusion (2.2) satisfies all the conditions of Definition 2.2. Theorem 3.2 is proved. □

Consider the system of difference equations

$$x(s+1) = B(s)x, \quad B(s) \in \Omega(s), \quad \Omega(s+M) = \Omega(s), \tag{3.4}$$

where $s = 0, 1, \ldots$, $x \in R^n$, $M \in N$.

The transition matrix of system (3.4) is the matrix $\Pi_B(s, s_0)$ connecting the solution $x_B(s, s_0, x_0)$ and x_0, i.e., satisfying the equalities $x_B(s, s_0, x_0) = \Pi_B(s, s_0)x_0$ and $\Pi_B(s_0, s_0) = E_n$, $s \geq s_0$, where E_n is the unit matrix of order n.

From the equality

$$\rho(s, s_0) = \max_{B(s)\in\Omega(s)} \|\Pi_B(s, s_0)\|^2, \; s \geq s_0,$$

it follows

Corollary 3.1:

Inclusion (2.2) is uniformly asymptotically stable if and only if the transition matrix $\Pi_B(s, s_0)$ *of system (3.4) satisfies the condition* $\lim_{s\to+\infty} \|\Pi_B(s, s_0)\| = 0$, *uniformly in* $s_0 \geq 0$ *and* $B(s) \in \Omega(s)$.

The equivalence of the uniform asymptotic stability and the uniform exponential stability for inclusion (2.2) is established by the following theorem.

Theorem 3.3:

Inclusion (2.2) is uniformly asymptotically stable if and only if inclusion (2.2) is uniformly exponentially stable.

Proof. The sufficiency follows directly from Definitions 2.3 and 2.4, and only necessity needs to be proved.

To prove the necessity we use the statement of Theorem 3.2. Let us show that the uniform exponential stability of inclusion (2.2) follows from condition (3.2).

For solutions of inclusion (2.2) we have

$$\|x_B(s, s_0, x_0)\|^2 = \|x_0\|^2 \|x_B(s, s_0, x_0/\|x_0\|)\|^2 \leq \rho(s, s_0)\|x_0\|^2, \tag{3.5}$$

for all $\{s_0 \geq 0,\ x_0 : x_0 \neq 0,\ B(s) \in \Omega(s)\}$, where the function $\rho(s, s_0)$ is defined in (3.1).

Note that solutions of inclusion (2.2) satisfy the equality

$$x_B(s_2, s_1, x_B(s, s_0, x_0)) = x_B(s_2, s_0, x_0)$$

for all $s_2 \geq s_1 \geq s_0$. Let $s = s_0 + kS + m$, where $k = [(s - s_0)/S] \geq 0$, $0 \leq m < S$ ($[z]$ denotes the integer part of z).

Using (3.5), we obtain

$$\|x_B(s_0 + kS + m, s_0, x_0)\|^2 = \|x_B(s_0 + kS + m, s_0 + (k-1)S + m, x_B(s_0 + (k-1)S + m, s_0, x_0))\|^2 \leq$$

$$\leq \rho(s_0 + kS + m, s_0 + (k-1)S + m) \cdot \|x_B(s_0 + (k-1)S + m, s_0, x_0))\|^2 \leq$$

$$\leq \rho(s,s-S)\ \rho(s-S,s-2S)\cdot\ldots\cdot\rho(s-(k-1)S,s_0+m)\cdot\left\|x_B(s_0+m,s_0,x_0)\right\|^2. \tag{3.6}$$

Since inclusion (2.2) is uniformly asymptotically stable, relation (3.2) holds. Therefore, for any $\eta>0$ there exists $S(\eta)$, such that

$$r(s-(j-1)S,s-jS)<\eta=\exp(-\chi)<1 \quad (\chi>0).$$

for each of the factors $\rho(s-(j-1)S,s-jS),\ j=\overline{1,k}$ in (3.6).

Therefore, for any $k=0,1,2...$

$$\left\|x_B(s,s_0,x_0)\right\|^2\leq\exp(-\chi k)\left\|x_B(s_0+m,s_0,x_0)\right\|^2. \tag{3.7}$$

Taking into account that $s(s,s_0)=1$, from (3.5) we obtain the estimate

$$\left\|x_B(s_0+m,s_0,x_0)\right\|^2\leq r(s_0+m,s_0)\left\|x_0\right\|^2\leq\beta_0\left\|x_0\right\|^2, \tag{3.8}$$

where $\beta_0=\max\limits_{0\leq m\leq S}r(s_0+m,s_0)\geq 1$.

From (3.7) and (3.8) it follows

$$\left\|x_B(s,s_0,x_0)\right\|^2\leq\beta_0\left\|x_0\right\|^2\exp(-\chi k),\ k=0,1,2,...$$

Since $k=[(s-s_0)/S]$, we have

$$\left\|x_B(s,s_0,x_0)\right\|\leq\beta\left\|x_0\right\|\exp(-\alpha(s-s_0)),$$

for solutions of inclusion (2.2), where $\beta=\sqrt{\beta_0e^{\chi}}\geq 1$, $\alpha=\chi/(2S)$, and the numbers $\alpha>0$ and $\beta\geq 1$ do not depend on $s_0\geq 0$, x_0 and $B(s)\in\Omega(s)$. Consequently, under Theorem 3.3, inclusion (2.2) is uniformly exponentially stable. Theorem 3.3 is proved. □

It follows from Theorem 3.2 that for uniformly asymptotically stable inclusion (2.2) there exists a number $S\in N$, independent of (s_0,x_0), such that $\rho(S,s_0)<1$. Using this property in the proof of Theorem 3.3 allows us to establish the uniform exponential stability of inclusion (2.2). Therefore, the following statement is true.

Corollary 3.2:

For inclusion (2.2) to be uniformly asymptotically stable, it is necessary and sufficient that there exists the number $S\in N$, such that $\rho(S,s_0)<1$.

4. EXAMPLES

Example 4.1:

The problem of absolute stability was solved in [6] for nonlinear nonstationary discrete control systems with a periodic linear part described by the equations

$$x(s+1)=A(s)x(s)+\sum_{j=1}^{m}b^j(s)\varphi_j(\sigma_j(s,x(s)),s), \tag{4.1}$$

where

$$\sigma_j(s,x) = \langle c^j(s), x\rangle = \sum_{i=1}^{n} c_i^j(s)x_i, \varphi_j(0,s) \equiv 0, j = \overline{1,m},$$

$x = (x_i)_{i=1}^n, x \in R^n$ is an n-dimensional vector, characterizing the deviation of the system from the zero solution $x \equiv 0$, $A(s)$ is a square matrix of order n, $b(s), c(s), j = \overline{1,m}$ are n-dimensional vectors, $s = 0,1,...$ is discrete time, the brackets $\langle .,. \rangle$ denote the inner products. It is assumed that the matrix and the vectors $b^j(s), c^j(s)$ are bounded and satisfy the periodicity conditions

$$A(s+M) = A(s), \ b^j(s+M) = b^j(s), \ c^j(s+M) = c^j(s), \quad M \in N$$

for each $s = 0, 1,...$ and $j = \overline{1,m}.$ It is also assumed that the nonlinear functions $\varphi_j(\sigma_j(s,x),s), j = \overline{1,m},$ specifying the characteristics of nonlinear nonstationary elements are defined for all $x \in R^n$, $s \geq 0$ and satisfy the standard sector-type constraints.

System (4.1) is equivalent to periodic selector-linear difference inclusion (2.2), where the multivalued function $F(s,x)$ $(F(s+M,x) \equiv F(s,x))$ is defined in each point (s,x) ($s \geq 0, x \in R^n$) by the relation

$$F(s,x) = \left\{ y : y = A(s)x + \sum_{j=1}^{m} b^j(s)\mu_j \langle c^j(s), x\rangle, \ \mu_j \in [k_{1j,} k_{2j}], \ \ j = \overline{1,m} \ \right\}. \tag{4.2}$$

Here and in the following example, equivalence is understood in the sense of coincidence of the sets of solutions of the considered system and the corresponding inclusion with the same initial conditions.

Example 4.2:

Consider the equation with real parameters a и b

$$\frac{d^2z}{dt^2} + (a^2 + bf(t))z = 0,$$
$$a \in I_1, \ b \in I_2, \ f(t+T) \equiv f(t), \ t \geq 0, \ T > 0, \tag{4.3}$$

where $f(t)$ is an integrable piecewise continuous periodic function, $I_1 = [0, a_1], \ I_2 = [b_1, b_2]$. An equation of this kind was first considered by the American mathematician and astronomer George Hill for the case when $f(t)$ is an even periodic function with period π, in connection with the study of the average motion of the lunar perigee. Later, the equation was studied in a large number of publications, mainly of an applied nature. A special case of Hill's equation is the equation that was obtained by the French mathematician and astronomer Emile Mathieu to describe vibrations of an elliptical membrane.

Let’s introduce the notation

$$x_1 = z, \ x_2 = \frac{dx_1}{dt} = \frac{dz}{dt}, \ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \ A(t,a,b) = \begin{pmatrix} 0 & 1 \\ -a^2 - bf(t) & 0 \end{pmatrix},$$

and rewrite equation (4.3)

$$\frac{dx}{dt} = A(t,a,b)x,\; A(t+T) \equiv A(t),$$

$$a \in I_1,\; b \in I_2,\; t \geq 0,\; T > 0,\; x \in R^2. \tag{4.4}$$

Consider the discrete analogue of system (4.4)

$$x(s+1) = A(s,a,b)x(s),\; A(s+M) \equiv A(s),$$

$$a \in I_1,\; b \in I_2, M \in N,\; x \in R^2, \tag{4.5}$$

where $s = 0,1,...$ is discrete time.

System (4.5) is equivalent to periodic selector-linear difference inclusion (2.2), where the multivalued function $F(s,x)$ $(F(s+M,x) \equiv F(s,x))$ is defined in each point $(s,x), x \in R^2$ by the relation

$$F(s,x) = \{y : y = A(s,a,b)x,\; a \in I_1,\; b \in I_2\}.$$

5. CONCLUSION

Periodic difference inclusions are considered. It is proved that under some restriction on the initial conditions, the convergence of solutions to zero is uniform. For periodic selector-linear difference inclusions the equivalence of the properties of uniform asymptotic stability and uniform exponential stability of the zero solution is established. Using the variational method the necessary and sufficient condition for the uniform asymptotic stability in the form of the limit relation has been obtained. Examples of systems leading to the consideration of periodic difference inclusions are presented.

The results obtained can be used in the analysis of stability of control systems with periodic parameters, in particular, servomechanisms whose elements operate on alternating current, control systems with amplitude-frequency modulation, systems used to solve problems associated with the study of large electric power systems in the presence of forced oscillations.

Further investigation of the inclusions considered in this paper may be related to the search for parametric classes of Lyapunov functions that establish necessary and sufficient conditions for asymptotic stability.

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