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Choosing Directions for Investments in the Development of Companies Under Uncertainty

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Abstract: The problem of choosing investment decisions of companies in the oligopoly market in conditions of uncertainty in demand for products is considered. Investment decisions include the choice of the size and ratio of funds for the implementation of projects of two types: projects to expand production and, accordingly, to increase the supply of products on the market; as well as projects aimed at reducing production costs, which only affect the profitability of production and the free cash flow of companies. We propose an approach based on the joint use of the DCF model and algorithms that describe the investment behavior of companies in the market. The model takes into account the relationship between the choice of investment decisions by companies and the dynamics of the market price. The solution of the problem is reduced to the analysis of a matrix game in which the payoff matrix is formed as a result of numerical simulation. An illustrative example of using the proposed approach is given.

Keywords: investment decisions, oligopoly, demand uncertainty, company behavior model

1. INTRODUCTION

We consider the problem of analyzing and choosing investment strategies for companies under conditions of uncertainty in market demand. We are looking at oligopolistic markets, where two or more companies compete with each other for market share and therefore profit share. Companies can choose different investment strategies knowing that the future dynamics of market demand is uncertain. Different investment strategies, depending on the implementation of the demand dynamics scenario, can lead to both profits and losses for the company. In addition, when setting the problem, an important factor of mutual influence of the investment strategies of companies and the dynamics of prices for products will be taken into account [2-4].

The investment strategy determines the amount of funds directed by the company to invest in expanding production and (or) reducing production costs. The cost of products (below the industry average or above the industry average) is of great importance in the competitive struggle. Consequently, an investment strategy aimed at reducing costs is often preferable to a production expansion strategy.

The modern theory of investments in conditions of uncertainty has been developing over the past twenty to twenty-five years and is associated with the development of analysis methods using the ideology of evaluating the value of real options of various types in continuous time. The fundamentals of the methodology of this approach are outlined in a number of publications [6, 10, 11], which had a great influence on the development of this area of research. Currently, it is characterized by a wide arsenal of methods and many problems statements [5-11, 13].

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For most variants of these problems, qualitative or quantitative results were obtained under the following fairly strong assumptions:

1. It is assumed that investments occur instantly and immediately after that there is an infinite cash flow, which is the main factor of uncertainty. In a number of works, the uncertainty of the cash flow is determined by the stochastic demand and supply of products on the market.
2. It is also assumed that cash flow or demand behaves as a random process of a certain kind. The overwhelming majority of works use geometric Brownian motion (GBM) to model this uncertainty, which was originally used in the Black-Scholes-Merton option pricing model in financial markets, and consists of the following:

A random process St is a geometric Brownian motion (GBM) if it satisfies the following stochastic differential equation [12]:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

In this model, the parameter μ is a non-random trend, σ is the degree of volatility, and Wt is the standard Wiener process (Brownian motion). Note that this approach assumes that the parameters μ and σ in the stochastic equation are constant.

This approach, according to its supporters, describes well the volatility of asset prices in financial markets and can be applied with great caution to the valuation of real assets, in particular, to the assessment of investment projects and investment strategies. There is some pessimism regarding the practical value of this approach [12]. In addition, these models provide only a first approximation to the complex processes taking place in modern financial and especially commodity markets.

It should be emphasized that in equation (1) the parameter μ (non-random trend) is considered constant and does not depend on time. This assumption significantly reduces the realism of modeling the dynamics of demand in the market, the rate of change of which can also vary significantly over time if we consider time intervals of several years, which is a characteristic feature of investment projects in real sectors of the economy. And, of course, the function μ (t) is also unknown in advance.

For large investment projects, the forecasting horizon is usually 10-15 years. Demand shocks can occur during this period, when demand can change dramatically. For example, “demand shocks” in the markets occurred in 2008-2009. (World economic crisis) and in 2014-2015. (Falling oil prices). For example, the demand for metal on the Russian market fell by 20-25% in 2009 and began to partially recover only by 2010-2011. In 2015, domestic demand for metal also decreased by 10-15% due to a significant decrease in production and sales in the automotive industry (-30%) and a decrease in the construction industry.

Another important note. For the purposes of evaluating the investment decisions of companies, the parameter μ (t) is more important than the second stochastic term. The influence of the second term in formula (1) decreases if we take into account that time-averaged integral characteristics are used to assess the efficiency of investment decisions, such as, for example, the NPV indicator.

Regarding the first assumption, it is well known that the investment process of a company is continuous. The investment effect (cash flow) occurs with a lag, which depends on the duration of the investment phase of the project. In addition, cash flow can change significantly over different periods of time, including under the influence of random demand, the results of the choice of investment strategies of companies and product prices.

As shown, there is a significant gap in the degree of realistic modeling of investment processes between theoretical works based on the methodology of real options and traditional methods of modeling investment projects based on the DCF methodology. DCF models more realistically allow simulating cash flows at different stages of the implementation of investment projects, which explains their wide application in practice.

However, these methods do not work well in the face of uncertainty in the model input data, including price and demand volatility associated with company competition in the market. This

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is one of the main arguments against them on the part of the proponents of the real options approach [6].

The article attempts to mitigate the disadvantages of both approaches. For this, it is proposed to use jointly methods of scenario modeling of market uncertainty and aggregated DCF models, which are supplemented by the behavioral models of companies making investment decisions based on changing market information.

2. THE MODEL

Consider a market with N companies. Each company can make investment decisions in accordance with the chosen strategy and based on information from the market. The criterion for the success of the company's investment strategy is the NPV indicator (total discounted free cash flow of the company for the forecast period).

As noted earlier, the choice of investment decisions is significantly influenced by the forecast of demand and prices for products. We will consider a situation of high demand volatility, including when market demand can change the trend several times during the forecast period. This situation is the most difficult to assess and analyze the effectiveness of investment decisions. The difficulty lies in the fact that companies cannot predict changes in demand for the entire forecast period and are limited only to assessing the trend that is observed during the period in which the company's investment budget is formed.

The question arises - what should be the investment strategy of companies in the face of uncertainty in demand and uncertainty in the behavior of competitors? The situation becomes even more complicated if we take into account the influence of companies' investment activity on the dynamics of market parameters. Excessive investment activity of companies, as a rule, leads to the emergence of "extra" production capacity and, during periods of declining demand, to a significant decrease in the price of products [2, 3].

We consider a situation where campaigns at the beginning of each period form investment budgets based on some rational rules, using the results of analyzing trends in the dynamics of demand and product prices.

In this paper, the term "choice of an investment strategy" includes: the choice of the company's financial resources allocated for investment, and the choice of the investment direction, which determines the ratio of funds allocated for the implementation of projects of two types:

- projects to expand production and, accordingly, increase the supply of products on the market;
- projects aimed at reducing production costs, which only affect the profitability of production and the free cash flow of companies.

Consider a time period (forecast period) equal to T periods, $t = \overline{1,T}$. $NCF_i(t)$ - free cash flow of company i ($i = \overline{1,N}$) in period t, equal to the company's net profit for this period minus investments, and is calculated by the formula:

$$NCF_i(t) = (P(t) - C_i(t)) \cdot B_i(t) \cdot (1-p) - I_i(t) \tag{2}$$

Where: $P(t)$ - the market price of products in the period t. In each period of time, the price is formed based on the ratio of demand for products $D(t)$ and the total supply of manufacturers $S(t)$. $S(t)$ is determined in each period t as $S(t) = \sum_{i=1}^{N} S_i(t)$, where $S_i(t)$ is the production capacity of company i.

Then $P(t) = P(0) \cdot (1 + \gamma \cdot (\frac{D(t) - S(t)}{D(t)})$, where the parameter γ is the price elasticity on the value of the excess of demand over supply. Here $P(0)$ is the market price at the beginning of the forecast period (initial conditions). If there $D(t) - S(t) \geq 0$ is a shortage of supply in the market, and the price rises, otherwise, there is an excess of supply and, accordingly, the price falls.

It should be noted that $D(t)$ may also depend on price dynamics. An increase $P(t)$ can lead to a decrease $D(t)$, which is taken into account by introducing negative feedback into the model. The degree of price influence on demand is specified through the parameter of demand elasticity in relation to changes in the market price. Further, markets with inelastic demand are considered, which, in particular, include the metallurgical industry.

Let, further, $B(t)$ be the total sales in the market in period t, which is calculated in the model using the following formula: $B(t) = \min\{D(t); S(t)\}$. Suppose that the capacity utilization of all companies is the same, then the sales revenue of company i is calculated as follows:

$$B_i(t) = B(t) \cdot \frac{S_i(t)}{S(t)}$$

$C_i(t)$ is the cost of production of company i in period t, p is the income tax rate. Note that all the quantities included in the calculation formula $NCF_i(t)$ depend on random demand $D(t)$ and the choice of investment strategies of companies that determine the dynamics of their production capacity $S_i(t)$ and production costs $C_i(t)$.

Next, we will consider a model that allows us to evaluate the effectiveness of investment strategies of companies, taking into account the market uncertainty of demand. The general structure of the developed model for the duopoly market is shown in Fig. 1. Demand dynamics $D(t)$. $D(t)$ - exogenous variable of the model, the graph, which sets various external macroeconomic scenarios in relation to the model.

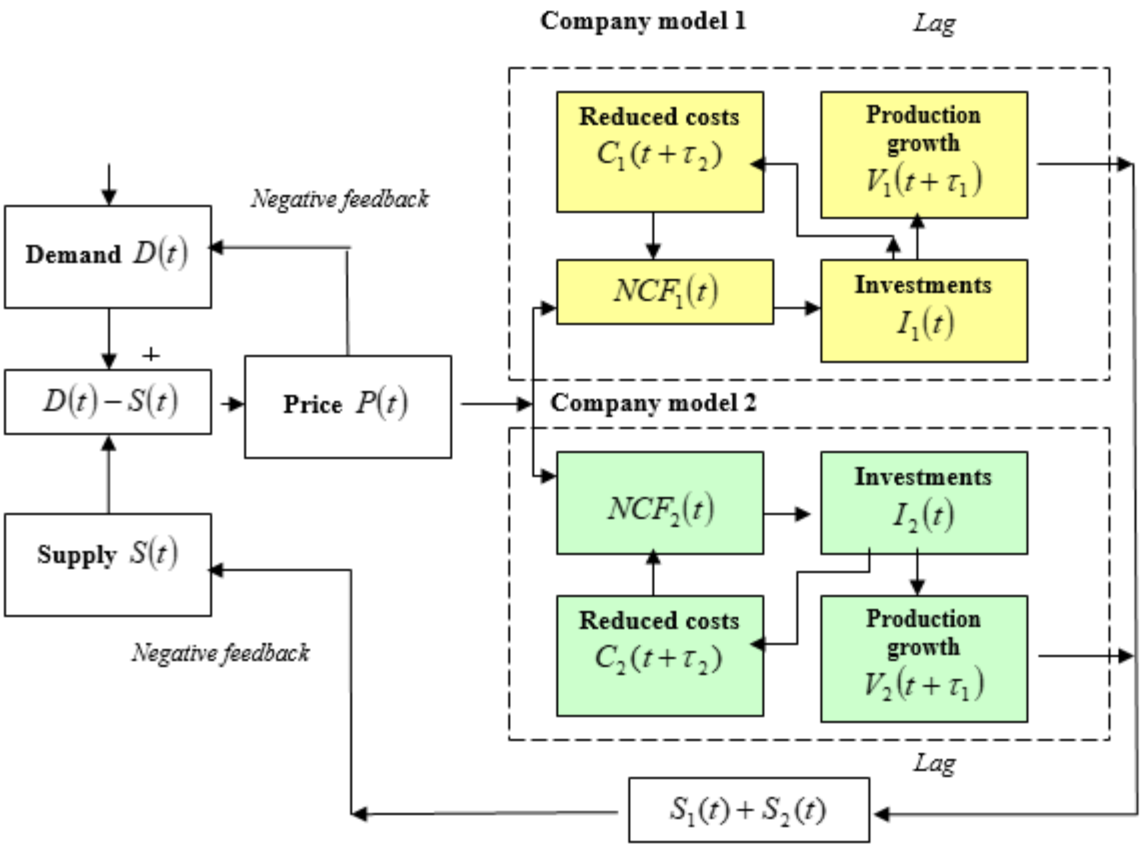

Fig. 1. Model structure

2.1 Company behavior model

We assume that companies make investment decisions in the face of market volatility and uncertainty in the dynamics of demand for companies' products. Companies can observe in each period t only the change in their financial indicators (net profit, product price, sales volume) and (or) predict their change for the next several periods. The model allows varying the depth of a "reliable" forecast of market dynamics available to market participants. This makes it possible to take into account the factor of "foresight" of companies in the analysis. Obviously, in case of low market volatility, the depth of the "reliable" forecast can be increased. At the beginning of each period t, the company makes an investment decision based on this available information in accordance with some predetermined algorithm, which will be described below.

Let further, if $P(t)-P(t-1)>0$ or $B_i(t)-B_i(t-1)>0$ (which signals the company about an upward trend in the market), then part of the company's net cash flow accumulated over period t in a share equal α_i to this can be directed to investments in its development - $I_i^*(t)$. The value α_i determines the investment activity of the company i. The higher the value (share), the higher its investment activity. Thus $I_i^*(t)=\alpha_i\cdot\sum_{t=1}^{t-1}NCF_i(t)$.

As noted earlier, a company can direct total investments in projects of two types: projects aimed at increasing production capacity (projects of the first type) and projects aimed at reducing costs (projects of the second type) in a certain ratio α_i^1 and α_i^2, ($\alpha_i^2=1-\alpha_i^1$).

The quantities α_i, α_i^1 and α_i^2 are the parameters of the model that companies can choose based on their forecasts of market dynamics.

Thus, the company chooses investments $I_i(t)$ based on the analysis of actual data and a possible forecast of market dynamics. $I_i(t)$ cannot exceed $I_i^*(t)$, which is determined, in turn, through a variable parameter of investment activity. $I_i(t)=I_i^1(t)+I_i^2(t)$, where: $I_i^1(t)$ and $I_i^2(t)$ investments in projects of the first and second types, respectively, which are determined in accordance with some algorithms described below.

2.2 First type investment

Company i invests in projects of the first type according to the following algorithm:

If during the period t there is an upward trend in the market (supply-demand> 0), then the company invests in projects of the first type as follows:

$I_i^1(t)=\min\{\alpha_i^1\cdot I_i^*(t),I_{np}^1\}$, where $I^1{}_{np}$ is the maximum allowable level of investments in projects of the first type for the period. If there is a downward trend in the market during period t, then this signals the company about the appearance of excess production capacity and, in accordance with this, the company does not invest in projects of the first type, i.e., $I_i^1(t)=0$.

Suppose there is a lag τ_1 between the investment period and the period of increasing production capacity - $V_i(t)$. Let also the value E_1 characterize the increase in production capacity per unit of investment. Then $V_i(t)=E_1\cdot I_i^1(t-\tau_1)$, production capacity is calculated using a recurring formula $S_i(t)=S_i(t-1)+V_i(t)$. At t = 1, the initial production capacity of the company i - $S_i(0)$ is set.

2.3 Second type investment.

Let us consider the influence of the company's investment decisions on the dynamics of changes in the cost of production. The cost of production in period t is calculated using the following formula:

$$C_i(t) = C_i(t-1) - E_2(t) \cdot I_i^2(t-\tau_2) \tag{3}$$

Where: $E_2(t)$- the specific efficiency of investments in projects to reduce production costs, τ_2 - the lag between the investment period and the period of the corresponding change in the cost. For t = 1, set $C_i(0)$ is the production cost at the beginning of the forecast period.

The value $E_2(t)$ characterizes the reduction in the cost of production per unit of investment in projects of the second type. The calculations are based on the "Decline in investment efficiency" model. In accordance with this model, $E_2(t)$ it decreases as the cost of production changes $C_i(t)$ and approaches a certain threshold value C_{np}. C_{np} - assessment of the maximum possible reduction in production costs, upon reaching which the investment efficiency becomes equal to zero - $E_2(t) = 0$.

$$E_2(t) = E_2(0) \cdot (1 - \frac{C_i(0) - C_i(t)}{C_{np}}) \tag{4}$$

The company invests in projects of the second type according to the following algorithm: $I_i^2(t) = \min\{I_i^*(t) \cdot (1-\alpha_i^1) \cdot \frac{E_2(t)}{E_2(0)}, I_{np}^2\}$, where $I^2{}_{np}$ is the maximum allowable level of investments in projects of the second type for the period. In accordance with the described algorithm, if starting from period t $E_2(t)$ becomes equal to zero, then the amount of investment will also $I_i^2(t)$ become equal to zero.

In accordance with this algorithm, the volume of investments of company i in period t depends on the accumulated cash flow for this period, the selected parameters α_i and α_i^1, as well as on the ratio of the current investment efficiency indicator to the investment efficiency at the beginning of the period. forecast period.

3. THE PROBLEM OF CHOOSING AN INVESTMENT STRATEGY

The presented model makes it possible to calculate the free cash flow of companies based on the choice of parameters of investment strategies $(\alpha_i, \alpha_i^1, i = \overline{1,N})$ and scenarios of market demand dynamics ψ.

Let be $NCF_i(t)$ the free cash flow of company i $(i = \overline{1,N})$, which depends on the choice of investment strategies by all companies and the implementation of the scenario of market demand dynamics. Then the indicator of the effectiveness of the chosen investment strategy is calculated as the difference between the free cash flow of the company implementing the investment strategy with parameters α_i, α_i^1 and the free cash flow of the company in the absence of investments, i.e., $\alpha_i = 0, \alpha_i^1 = 0$.

$$NPV_i = \sum_{t=1}^{t=T} (NCF_i(t, \alpha_i, \alpha_i^1) - NCF_i(t,0,0)) \cdot \frac{1}{(1+d)^t} \tag{5}$$

Where: d is the discount rate.

Companies make decisions to invest in expanding production or reducing production costs in accordance with the rules described in the previous sections. For each scenario of market demand dynamics ψ, companies choose investment strategies $(\alpha_i, \alpha_i^1, i = \overline{1, N})$ that maximize (5), taking into account the possible choice of investment strategies of competitors.

In the above formulation of the problem, each company solves its own maximization problem (5). The sought-for task variables are parameters α_i and α_i^1. The difficulty in solving this problem is that the dependence $NCF_i(t, \alpha_i, \alpha_i^1)$ cannot be specified in the form of an explicit analytical expression on α_i and α_i^1. The value of the optimality criterion (5) is calculated each time for given α_i and α_i^1 for all agents using the simulation model described in the previous section.

This problem belongs to a class of optimization problems that is quite difficult to solve, in which the values of the criterion and constraints are set using a simulation model. Standard mathematical programming methods are not applicable here.

A grid search algorithm for finding a solution to the problem is proposed, which consists in discretizing the set of solutions. For each point, a simulation experiment is carried out, and on the basis of the data obtained, matrices of criteria values are constructed for each agent. The general solution of the problem for N agents must satisfy the Nash equilibrium conditions (the saddle point of function (5) in the space of the sought variables). The final stage of the search for a solution is reduced to the analysis and search for a solution to the game, which is described below.

The problem can be reduced to the study of a model of a one-step continuous non-zero sum game, in which the payoff functions of players (companies) are specified by a simulation model. Therefore, numerical modeling methods will be used as the main method for studying this problem.

Note that the set of possible investment strategies for each company coincides with the set of points in the unit square on the plane. Without loss of generality, one can consider a finite set of strategies using, for example, grid search methods. Let, further, for greater clarity and simplicity of presentation, companies use several basic investment strategies. Consider, for example, the following set of strategies.

Strategy 1 ($\alpha_i = 1, \alpha_i^1 = 1$). Invest in production expansion (intensive development option). This strategy allows you to increase production volumes and, in a growing market, leads to an increase in sales and, accordingly, cash flow. In a falling market, this strategy leads to a decrease in the utilization of production equipment and an increase in production costs and, accordingly, a decrease in cash flow.

Strategy 2. ($\alpha_i = 1, \alpha_i^1 = 0$). Invest in lower production costs (extensive development option). This strategy allows you to reduce production costs without increasing production. In a growing market, this leads to maintaining sales and, accordingly, increasing cash flow by increasing profitability. In a falling market, this strategy allows you to maintain the amount of cash flow due to the lower "break-even point".

Often, companies use more cautious strategies in which the estimated volume of investment is evenly distributed between investments of the first and second types ($\alpha_i^1 = 0{,}5$). At the same time, only investment activity changes, which depends on the predicted dynamics of the market situation. As a rule, with the expected improvement in market conditions, companies increase their investment activity (α_i).

Strategy 3. ($\alpha_i = 1, \alpha_i^1 = 0{,}5$). High investment activity

Strategy 4 ($\alpha_i = 0{,}5, \alpha_i^1 = 0{,}5$). Moderate investment activity

Strategy 5 ($\alpha_i = 0, \alpha_i^1 = 0{,}5$). Low investment activity.

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Next, we'll look at the duopoly market. Let the set of investment strategies of company 1 ($k = \overline{1,K}$) and company 2 ($j = \overline{1,J}$). Based on the results of a series of calculations for each scenario, it is possible to construct companies' payoff matrices $NPV^1_{k,j}, NPV^2_{k,j}$

In this formulation, the solution of the problem is reduced to the analysis of a bimatrix game with payoff matrices $NPV^1_{k,j}, NPV^2_{k,j}$. This problem has been well researched. As is known, the condition for the existence of at least one Nash equilibrium point in pure strategies (k_0, j_0) is the fulfillment of the following inequalities:

$$NPV^1_{k_0 j_0} \geq NPV^1_{kj_0}, k = \overline{1,K} \tag{6}$$

$$NPV^2_{k_0 j_0} \geq NPV^2_{k_0 j}, j = \overline{1,J} \tag{7}$$

If such a point exists, then this is considered a solution to this problem. The possibility of obtaining a solution (Nash equilibrium point) of a bimatrix game in pure strategies, as a rule, is not guaranteed and depends on the properties of the matrix $NPV^1_{k,j}, NPV^2_{k,j}$. The method for solving this problem includes carrying out a series of numerical calculations on a simulation model, constructing a payment matrix, analyzing it, and finding a solution.

As an example of using the proposed approach, the following are the results of calculations and analysis of solutions to the problem for the case of three investment strategies and two scenarios of market demand dynamics. It will be shown that in many cases there are Nash equilibrium points in pure strategies; the analysis of game solutions allows us to draw a number of conclusions that are interesting for practice.

The result of this stage of the analysis of the investment strategies of companies is the answer to the following question: if the scenario ψ is implemented, then what should be the companies' strategies that are optimal in the sense of criterion (5). The solution to the problem assumes that both companies consider the most likely implementation of the same scenario of market demand dynamics.

4. MODELING AND ANALYZING RESULTS

This section provides an illustrative example of the application of the proposed approach. Consider the duopoly market in the steel industry. A simplified situation is considered: there are two companies on the market that produce one type of metal products.

The following model parameters were used in the calculations:

It is assumed that in the period t = 0 supply and demand in the market are balanced, i.e., B (0) = D (0) = S (0), and equal to 10 million tons per year. P (0) is the market price of the product, equal to USD 500 per tonne; C (0) - the cost of producing one ton of products is USD 400.

The parameter of price elasticity γ is equal to 0.5. This means that if the value of the imbalance deviates in the period t from the equilibrium value (in the period t = 0) by b%, respectively, the price of the product will change in relation to its equilibrium value by 0.5b% (with the same sign).

$E_1(t)$=0,03, i.e., with an investment of US \$ 100 million, the company's production capacity is increased by 3.0 thousand tons over the period. τ_1 is equal to 2 years, which corresponds to the average duration of the implementation of investment projects in metallurgy aimed at increasing production capacity. $I^1{}_{np}$ - the maximum allowable volume of investments in projects of the first type for the period was adopted at the level of USD 100 million per year.

E2 (0) = 0.015, i.e., with an investment of USD 100 million, the production cost of 1 ton of products is reduced by USD 1.5. The value of τ_2 in the calculations is taken to be 1 year.

These parameters of the model were obtained on the basis of an analysis of data on the implementation of investment projects in a large metallurgical company (NLMK Group) and, naturally, reflect the assessment of the averaged values of these parameters. This information

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is presented in detail in [1]. For other sectors of the economy, these parameters of the model should be revised.

The forecast period from 2020 to 2035 is considered. Let both companies have the same initial capacity (5 million tons of products per year).

Scenario 1 (steady growth in demand). In accordance with this scenario, market demand for products throughout the forecast period grows at a steady 4% per year (Fig. 2). The payoff matrix is presented in Table 1.

Table 1. Payoff Matrix (Scenario 1)

	j=1	j=2	j=3
k=1	**7120\7120**	**8097**\6414	**9221**\4262
k=2	6414**8097**	7481\7485	9056\4801
k=3	4262**9221**	4801\9056	6965\6965

The first element of the matrix corresponds to the winning of the first player, and the second element corresponds to the winning of the second player. In the table, the maximum elements of the columns of the matrix of the first player and the maximum elements of the rows of the matrix of the second player are highlighted in bold. Analysis of the resulting matrix shows the presence of a single Nash equilibrium point, which corresponds to the choice of investment strategy 1 by both companies (k = 1 and j = 1). At this stage, the elements of the matrix of the first and second players are highlighted in bold.

Figure 2 shows the dynamics of the expected market demand for products for scenario 1 and the dynamics of the growth of production capacity and, accordingly, the supply from companies 1 and 2.

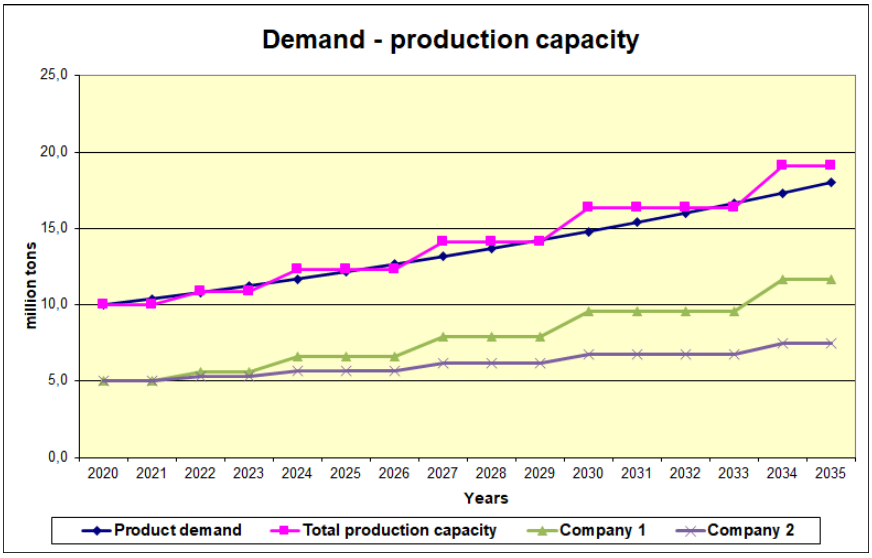

Fig. 2. Demand - production capacity (Scenario 1)

It is important to note that a solution to a bimatrix game can have multiple Nash points. This situation is illustrated by an example (table 1). At the first Nash point, the winnings of both players are the same and equal to USD 7120 million. However, if companies agree to choose a different point that corresponds to investment strategy 2 (k = 2 and j = 2), then the players' gains will amount to USD 7481 million, which exceeds their winnings at the first Nash point. The way out in such situations is the cooperation of the players. In real economic situations, competitors can interact with each other, entering into negotiations and concluding agreements,

often informal. If the payoff matrix of the game has a set of points that are Pareto optimal (i.e., others do not dominate), then such a set is called negotiated. Players can agree to jointly select a point from the negotiating set in a variety of ways, including various arbitration schemes.
The analysis of the game matrix (Table 1) illustrates the usefulness of such cooperation for both companies. If cooperation between players is impossible, it is more profitable for them to adhere to equilibrium strategies.

Scenario 2 (2020-2024 is the period of growth in demand, then, 2025-2026 is a sharp drop in demand, and, further, 2027-2035 is a slow steady recovery of demand to the level of 2024) (Fig. 3).
The payoff matrix for scenario 2 is shown in Table 2. In the table, the maximum elements of the columns of the matrix of the first player and the maximum elements of the rows of the matrix of the second player are highlighted in bold. Analysis of the resulting matrix shows the presence of a single Nash point, which corresponds to the choice of investment strategy 1 by both companies (k = 1 and j = 1). The elements of the matrix of the first and second players are highlighted in bold at this point.

Table 2. Payoff Matrix (Scenario 2)

	j=1	j=2	j=3
k=1	**4750\4750**	**4285**\3507	5764\2980
k=2	3507**4285**	4087\4087	**5941**\3715
k=3	2980\5764	3715**5941**	5098\5098

Note that the Nash point for both scenarios is the same. This suggests that in conditions of competition and lack of cooperation, regardless of the scenario, the most profitable strategy for both companies are the strategy of maximum investment activity.
The negotiation set consists of a single point that corresponds to the choice of investment strategy 3 by both companies (k = 3 and j = 3), which corresponds to the strategy of minimum investment activity. This set of player strategies provides them with the maximum payoff in case of their cooperation.
Figure 3 shows the dynamics of the expected market demand for products for scenario 2 and the dynamics of the growth of production capacities of company 1 and 2.

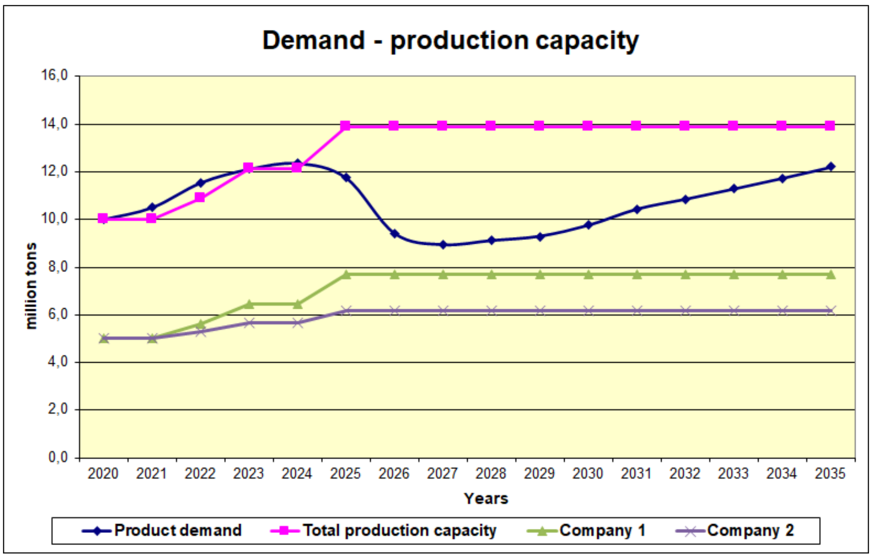

Fig. 3. Demand - production capacity (Scenario 2)

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5. CONCLUSION

The problem of analysis and selection of investment strategies of a company in the duopoly market, aimed at increasing their competitive advantages (increasing production capacities and reducing production costs), has been investigated.
The problem is reduced to the analysis of a bimatrix game, in which the payoff matrix is formed as a result of numerical simulation. The method for solving this problem includes carrying out numerical calculations on a simulation model, constructing a payoff matrix and analyzing it.
It is shown that in many cases there is a solution of a given game (Nash equilibrium point) in pure strategies. The analysis of decisions taking into account possible coalitions of players and various types of agreements between them made it possible to draw a number of qualitative conclusions that are interesting for practice.
It should be noted that the proposed approach to the study of the problem of choosing the investment strategies of companies opens up ample opportunities to study various modifications of this problem. For example, by varying the parameters of the model, one can consider various types of asymmetry in the market:

- one of the companies is the industry leader in terms of production volumes and market share (D1 (0)> D2 (0));
- one of the companies is the technology leader in the industry. Has lower production costs (S1 (0)> S2 (0));
- one of the companies has the best management, which translates into more efficient use of funds allocated for investments. E1 (0)> E2 (0).

The methodology presented in the article was used to study the investment strategies of companies in the rolled metal market, as well as in the oil market [3, 4].

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