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The Model of the Aggregated Consumer with Detailed Description of Consumer Financial Balance Using the Original Method of Relaxation of Complementary Slackness Conditions

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Abstract: This paper is complementary to a series of researches on aggregated consumer behavior and complementary slackness conditions application in linear nonautonomous optimal control type problems with mixed conditions. The model of an aggregated consumer is proposed. Consumer maximizes utility function in consumption and labor and chooses the values of its planned variables: loans and deposits in national and foreign currency, amount of real estate, and currency. The key feature of the proposed approach is a detailed description of all financial instruments that are available to the consumer.

Keywords: households, financial balance, complementary slackness conditions, macroeconomics, consumer financial instruments

1. INTRODUCTION

Aggregated consumer (or household) behavior can be modeled with various optimization problems starting from the relatively simple Ramsey growth model and ending with heavily modified DSGE models (see [3]-[6], [9], [10], [13]). However, the mainstream approach follows [11]. Consequently, stable foundations were built, which are rarely discussed. As a result, there are many similarities in various papers in consumer description. One of the disadvantages is that aggregated consumer (household) rarely has in its financial balance various financial instruments (see [1]-[3], [5], [6], [10]). Moreover, labor variable in utility function is not endogenous (if included) (see [1]-[3]). The main purpose of this research is to show how it is possible to describe an aggregated consumer in the macroeconomic model with the inclusion of a detailed description of consumer financial instruments. As various DSGE models show (see [3]-[5], [9], [10], [13]), the consumer usually operates with several bonds and very rare (or never) operates with a set of instruments: loans, deposits, foreign currency, real estate or others. In this paper, it is proved that such a description is possible due to the implementation of complementary slackness conditions (see [7], [8], [14]).

Nowadays, there is a discussion about different types of households: Ricardian and non-Ricardian (see [3]-[6], [13]). It complements the research vector on detailed consumer modeling.

Papers [12], [15] discuss utility function with endogenous labor. Multiplicative relation between CRRA consumption and labor functions shows high accuracy in the standalone aggregated consumer model. However, current empirical testing proved that additive relation is better if there is a need to include an aggregated consumer as part of the multi-block model. Also, additive relation between consumption and labor is used frequently (see [5], [6], [9], [10], [13]).

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2. AGGREGATED CONSUMER MODEL

The model describes the economic behavior of an aggregated consumer (household) that consumes goods and services, provides labor, manages its financial instruments: takes loans in national and foreign currency, makes deposits in national and foreign currency, buys foreign currency, performs operations with real estate and receives transfers.

By the time t the consumer takes loans in rubles $L(t)$ at an interest rate $r_l(t)$ and makes interest payments $r_l(t)L(t)$. The equation (2.1) describes the dynamics of loans on consumer balance:

$$\frac{d}{dt}L(t) = K(t) - \beta_l(t)L(t), \tag{2.1}$$

where $K(t) \geq 0$ is a flow variable for consumer loan operations and $\beta_l(t)$ is the consumer's average return frequency of loans.

Similarly, the consumer takes loans in foreign currency $vL(t)$ at an interest rate $r_{vl}(t)$ and makes interest payments in foreign currency $w_{vl}(t)r_{vl}(t)vL(t)$, where $w_{vl}(t)$ is the exchange rate. The equation (2.2) describes the dynamics of loans on consumer balance:

$$\frac{d}{dt}vL(t) = vK(t) - \beta_{vl}(t)vL(t). \tag{2.2}$$

In each moment t consumer saves $S(t)$ at an interest rate $r_s(t)$ and receives interest payments $r_s(t)S(t)$. The equation (2.3) describes the dynamics of deposits on consumer balance:

$$\frac{d}{dt}S(t) = V(t) - \beta_s(t)S(t), \tag{2.3}$$

where $V(t) \geq 0$ is a flow variable for deposit operations and $\beta_s(t)$ is the average frequency of making deposits by the consumer.

Similarly, by the time t the consumer saves $vS(t)$ at an interest rate $r_{vs}(t)$ and receives interest payments $r_{vs}(t)vS(t)$. The equation (2.4) describes the dynamics of deposits in foreign currency on consumer balance:

$$\frac{d}{dt}vS(t) = vV(t) - \beta_{vs}(t)vS(t). \tag{2.4}$$

The consumer performs operations with real estate $B(t)$ at an interest rate $r_b(t)$ and receives income $r_b(t)B(t)$. The equation (2.5) describes the dynamics of real estate amount on consumer balance:

$$\frac{d}{dt}B(t) = oB(t) - \beta_b(t)B(t), \tag{2.5}$$

where $oB(t)$ is a flow variable that describes money received due to operations with real estate and $\beta_b(t)$ is an average frequency of operations.

Additionally, the consumer can operate with foreign currency $vQ(t)$ at an exchange rate $w_{vq}(t)$. The equation (2.6) can describe the dynamics of foreign currency amount on consumer balance:

$$w_{vq}(t)\frac{d}{dt}vQ(t)=w_{vq}(t)\left(ovQ(t)-\beta_{vq}(t)vQ(t)\right), \tag{2.6}$$

where $ovQ(t)$ is the flow variable for foreign currency operations and $\beta_{vq}(t)$ is an average frequency of operations with foreign currency.

In addition to described relations, there are several exogenous boundaries. The consumer receives transfers from the government $Tr(t)\le Tr_0(t)$, where the consumer knows $Tr_0(t)$. Also, the consumer has other expenditures $OC(t)\ge OC_0(t)$, which help to consider unseen or missed expenditures data.

An aggregated household consumes $C(t)$ by the price $p_c(t)$ and earns $(1-\tau_r(t))\omega_r(t)R(t)$, where $\omega_r(t)$ is wage per capita, $R(t)$ - labor, and $\tau_r(t)$ - income tax rate. Considering $M(t)$ as cash that consumer has, it is possible to write the consumer financial balance, where the equation (2.7) describes cash dynamics:

$$\begin{aligned}&\frac{d}{dt}M(t)=-p_c(t)C(t)+(1-\tau_r(t))\omega_r(t)R(t)+\\&+r_s(t)S(t)-\frac{d}{dt}S(t)+r_{vs}(t)vS(t)-\frac{d}{dt}vS(t)-r_l(t)L(t)+\frac{d}{dt}L(t)-r_{vl}(t)vL(t)+\frac{d}{dt}vL(t).\\&-w_{vq}(t)\frac{d}{dt}vQ(t)+r_b(t)B(t)-\frac{d}{dt}B(t)+Tr(t)-OC(t)\end{aligned} \tag{2.7}$$

Additionally, there is a liquidity constraint:

$$\begin{aligned}&M(t)\ge \nu_s(t)\beta_s(t)S(t)+\nu_{vs}(t)\beta_{vs}(t)vS(t)w_{vs}(t)+\\&+\nu_l(t)\beta_l(t)L(t)+\nu_{vl}(t)\beta_{vl}(t)vL(t)w_{vl}(t)+\nu_b(t)\beta_b(t)B(t)+\nu_{vq}(t)\beta_{vq}(t)vQ(t)w_{vq}(t),\end{aligned} \tag{2.8}$$

where $\nu_s,\nu_{vs},\nu_l,\nu_{vl},\nu_b,\nu_{vq}$ are constants.

This model's consumption deflator $p_c(t)$, wage $\omega_r(t)$, income tax rate $\tau_r(t)$, interest rates $r_s(t),r_{vs}(t),r_l(t),r_{vl}(t),r_b(t)$, exchange rate $w_{vq}(t)$, transfers $Tr_0(t)$, and other expenditures $OC_0(t)$ are exogenous. Considering all the above, the consumer chooses the values of its planned variables (controls):

$$C(t),R(t),S(t),vS(t),L(t),vL(t),vQ(t),B(t).$$

The goal of the consumer is to maximize the total discounted utility from the consumption $C(t)$ and labor $R(t)$ as functional:
$\int_{t_0}^{T}U\left(\frac{C(t)}{C_U(t)},\frac{R(t)}{R_U(t)}\right)dt$, where $C_U(t)=\frac{\omega_r(t)R_U(t)}{p_c(t)}$ is potential consumption when all economically active population is employed and receives wage per capita $\omega_r(t)$. $R_U(t)$ is economically active population. It is a common technique to standardize labor using an exogenous variable. Moreover, basic macroeconomics models, e.g., the Ramsey model, consider labor as exogenous. Here labor is an employed population, and it is endogenous. Thus, aggregated consumer decision relies on the employed to economically active population ratio.

Utility function represents the discounted difference between CRRA functions of consumption and standardized labor:

$$U(t)=\left(\frac{\left(\frac{C(t)}{C_U(t)}\right)^{1-\beta}}{1-\beta}-\frac{\left(\frac{R(t)}{R_U(t)}\right)^{1-\alpha}}{1-\alpha}\right)e^{-\Delta t}. \tag{2.9}$$

A terminal condition is required to solve this type of problem with constraints:

$$aTercons(t_0)\le ATercons(T), \tag{2.10}$$

where

$$Tercons(t)=B(t)+L(t)+M(t)+S(t)+vL(t)+vQ(t)+vS(t). \tag{2.11}$$

a and A are vectors that consist of constants for each of the $Tercons$ variables at a time t_0 and T respectively.

The problem above is a linear nonautonomous optimal control type problem with mixed constraints. Enormous equations are omitted to keep narration clear. The solution process takes two steps:

1. Langrage functional is written where each constraint has an associated dual variable: non-negative – to inequality and arbitrary – to equality.
2. A system of sufficient conditions for optimality is constructed that consists of four types of equations:

- Initial differential equations, the initial conditions for which are assumed to be given;
- Conjugate differential equations for variables dual to the constraints of the problem;
- Transversality conditions that define terminal conditions for conjugate differential equations;
- Complementary slackness conditions for inequalities.

The example of a problem solution is described in [7], [8], [14]. Several techniques mentioned in studies are used here to perform the transition from continuous to discrete time and to implement complementary slackness conditions. Problem solution is a system of equations, where the variable $\rho(t)=-\frac{\frac{d}{dt}\xi(t)}{\xi(t)}$ is a part of equations written below. $\xi(t)$ is a dual variable to consumer financial balance. $-\frac{\frac{d}{dt}\xi(t)}{\xi(t)}$ will often appear in sufficient optimality conditions, making it logical to redesignate as $\rho(t)$: the rate of falling of dual variable or consumer profitability ratio. Equations (2.12) - (2.27) describe problem solution.

Consumer profitability ratio $\rho(t)$ depends on exogenous variables and parameters a_i, b_i, c_i, which are discussed later:

$$\rho(t) = \frac{H(t)}{D(t)}, \tag{2.12}$$

$$\begin{aligned}
H(t) &= -M(t-1) + \big(a_1 w_{vs}(t) g_w(t) - b_1 w_{vs}(t) g_w(t) + \nu_{vq} w_{vs}(t) + w_{vs}(t)(b_1 - 1)\beta_{vq}(t)\big) vQ(t-1) + \\
&+ \big((a_2 - 1) r_b(t) + (b_2 + \nu_b - 1)\beta_b(t)\big) B(t-1) + \\
&+ \big(a_5 w_{vs}(t) g_w(t) + b_5 w_{vs}(t) g_w(t) + w_{vs}(t)(a_5 + 1) r_{vl}(t) - w_{vs}(t)(b_5 - \nu_{vl} - 1)\beta_{vl}(t)\big) vL(t-1) + \\
&+ \big((a_6 + 1) r_l(t) + (-b_6 + \nu_l + 1)\beta_l(t)\big) L(t-1) + \\
&+ \big(a_3 w_{vs}(t) g_w(t) - b_3 w_{vs}(t) g_w(t) + w_{vs}(t)(a_3 - 1) r_{vs}(t) + w_{vs}(t)(b_3 + \nu_{vs} - 1)\beta_{vs}(t)\big) vS(t-1) + \\
&+ \big((a_4 - 1) r_s(t) + (b_4 + \nu_s - 1)\beta_s(t)\big) S(t-1)
\end{aligned}$$

$$\begin{aligned}
D(t) &= w_{vs}(t)(a_1 \nu_{vq} + a_1 - b_1) vQ(t-1) + (\beta_b(t) a_2 \nu_b + a_2 - b_2) B(t-1) + \\
&+ (-w_{vs}(t)\beta_{vl}(t) a_5 \nu_{vl} + a_5 w_{vs}(t) + b_5 w_{vs}(t)) vL(t-1) + (-\beta_l(t) a_6 \nu_l + a_6 + b_6) L(t-1) + . \\
&+ (a_3 w_{vs}(t)\beta_{vs}(t)\nu_{vs} + a_3 w_{vs}(t) - b_3 w_{vs}(t)) vS(t-1) + (\beta_s(t) a_4 \nu_s + a_4 - b_4) S(t-1)
\end{aligned}$$

Briefly, $\rho(t)$ is a result of the substitution of all model variables in consumer financial balance (2.7) after sufficient optimality conditions equation system is solved. [7], [8], [14] describe the detailed solution.

Consumption at a time t depends on consumption in the previous period and exogenous variables and parameters:

$$C(t) - C(t-1) = \left(-\frac{\Delta}{\beta} - \frac{p_c(t) - p_c(t-1)}{\beta p_c(t-1)} + \frac{(\beta - 1)\big(C_U(t) - C_U(t-1)\big)}{\beta C_U(t-1)} + \frac{\rho(t)}{\beta} \right) C(t-1). \tag{2.13}$$

Labor is similar to consumption and follows from a problem as an endogenous variable:

$$R(t) - R(t-1) = \left(\begin{array}{l} -\dfrac{\tau_r(t) - \tau_r(t-1) + \Delta\tau_r(t-1) - \Delta}{\alpha\big(\tau_r(t-1) - 1\big)} + \dfrac{\big(1 - \tau_r(t-1)\big)\big(\omega_r(t) - \omega_r(t-1)\big)}{\alpha\big(\tau_r(t-1) - 1\big)\omega_r(t-1)} + \\ + \dfrac{(\alpha - 1)\big(R_U(t) - R_U(t-1)\big)}{\alpha R_U(t-1)} + \dfrac{\rho(t)\xi(t)\big(\tau_r(t-1) - 1\big)}{\alpha\big(\tau_r(t-1) - 1\big)\xi(t)} \end{array} \right) R(t-1). \tag{2.14}$$

The following variables are described with the help of the complementary slackness conditions technique.

Loans in national currency:

$$L(t) = K(t) - \beta_l(t) L(t-1) + L(t-1). \tag{2.15}$$

Loans in foreign currency:

$$vL(t) = vK(t) - \beta_{vl}(t) vL(t-1) + vL(t-1). \tag{2.16}$$

Deposits in national currency:

$$S(t) = V(t) - \beta_s(t)S(t-1) + S(t-1)\,. \tag{2.17}$$

Deposits in foreign currency:

$$vS(t) = vV(t) - \beta_{vs}(t)vS(t-1) + vS(t-1)\,. \tag{2.18}$$

Real estate that consumer owns:

$$B(t) = oB(t) - \beta_b(t)B(t-1) + B(t-1)\,. \tag{2.19}$$

Foreign currency:

$$vQ(t) = ovQ(t) - \beta_{vq}(t)vQ(t-1) + vQ(t-1)\,. \tag{2.20}$$

The following equations describe flow variables.
Income received from operations with foreign currency:

$$\begin{gathered} ovQ(t) = \frac{\left(-p_c(t)C(t) + Tr_0(t) - OC_0(t) - \omega_r(t)R(t)\left(\tau_r(t)-1\right)\right)c_1}{w_{vq}(t)} + \\ + \left(\left(-a_1\nu_{vq} - a_1 + b_1\right)\rho(t) + b_1\beta_{vq}(t) + \frac{\left(w_{vq}(t) - w_{vq}(t-1)\right)\left(a_1 - b_1\right)}{w_{vq}(t-1)}\right)vQ(t-1). \end{gathered} \tag{2.21}$$

Income received from operations with real estate:

$$\begin{gathered} oB(t) = \left(-p_c(t)C(t) + Tr_0(t) - OC_0(t) - \omega_r(t)R(t)\left(\tau_r(t)-1\right)\right)c_2 + \\ + \left(\left(-\beta_b(t)a_2\nu_b - a_2 + b_2\right)\rho(t) + b_2\beta_b(t) + r_b(t)a_2\right)B(t-1). \end{gathered} \tag{2.22}$$

Consumer deposit operations in foreign currency:

$$\begin{gathered} vV(t) = \frac{\left(-p_c(t)C(t) + Tr_0(t) - OC_0(t) - \omega_r(t)R(t)\left(\tau_r(t)-1\right)\right)c_3}{w_{vs}(t)} + \\ + \left(\left(-a_3\beta_{vs}(t)\nu_{vs} - a_3 + b_3\right)\rho(t) + b_3\beta_{vs}(t) + r_{vs}(t)a_3 + \frac{\left(w_{vs}(t) - w_{vs}(t-1)\right)\left(a_3 - b_3\right)}{w_{vs}(t-1)}\right)vS(t-1). \end{gathered} \tag{2.23}$$

Consumer deposit operations in national currency:

$$\begin{gathered} V(t) = \left(-p_c(t)C(t) + Tr_0(t) - OC_0(t) - \omega_r(t)R(t)\left(\tau_r(t)-1\right)\right)c_4 + \\ + \left(\left(-\beta_s(t)a_4\nu_s - a_4 + b_4\right)\rho(t) + b_4\beta_s(t) + r_s(t)a_4\right)S(t-1). \end{gathered} \tag{2.24}$$

Consumer loan operations in foreign currency:

$$vK(t)=\frac{\left(-p_c(t)C(t)+Tr_0(t)-OC_0(t)-\omega_r(t)R(t)\left(\tau_r(t)-1\right)\right)c_5}{w_{vl}(t)}+ \\ +\left(\left(-a_5\beta_{vl}(t)\nu_{vl}+a_5+b_5\right)\rho(t)+b_5\beta_{vl}(t)-r_{vl}(t)a_5-\frac{\left(w_{vl}(t)-w_{vl}(t-1)\right)\left(a_5+b_5\right)}{w_{vl}(t-1)}\right)vL(t-1). \tag{2.25}$$

Consumer loan operations in national currency:

$$K(t)=-\left(-p_c(t)C(t)+Tr_0(t)-OC_0(t)-\omega_r(t)R(t)\left(\tau_r(t)-1\right)\right)\left(1-c_1-c_2-c_3-c_4-c_5\right)+ \\ +\left(\left(-\beta_l(t)a_6\nu_l+a_6+b_6\right)\rho(t)+b_6\beta_l(t)-r_l(t)a_6\right)L(t-1). \tag{2.26}$$

Consumer liquidity:

$$M(t)=\nu_s\beta_s(t)S(t-1)+\nu_{vs}\beta_{vs}(t)vS(t-1)w_{vs}(t)+\nu_{vl}\beta_{vl}(t)vL(t-1)w_{vl}(t)+ \\ +\nu_b\beta_b(t)B(t-1)+\nu_{vq}vQ(t-1)w_{vq}(t). \tag{2.27}$$

3. EMPIRICAL TESTING

The model was tested on quarterly data from Q1-2008 to Q1-2021. The restriction for the chosen period is data availability. The primary data sources are the Federal State Statistics Service (FSSS) and The Central Bank of Russia (CBR). The table below describes the connection between statistics and model variables :

Table 3.1. Model variables matching with data

Variable	Statistics	Source
$C(t)$	Final consumption expenditures, 2016 prices, billions of rubles	FSSS
$C_U(t)$	$C_U(t)=\frac{\omega_r(t)R_U(t)}{p_c(t)}$	Author's calculation using variables $\omega_r(t), R_U(t), p_c(t)$
$p_c(t)$	Final consumption cost deflator	FSSS
$R(t)$	Labor, million people	FSSS
$R_U(t)$	Economically active population, mil people	FSSS
$\omega_r(t)$	Wage rate per employee, thousand rubles/person	FSSS
$\tau_r(t)$	The personal income tax rate	FSSS
$Tr(t)$	Transfers	FSSS
$w(t)$	Nominal USD / RUB exchange rate at the end of the period	CBR
$g_w(t)$	$\frac{w(t)}{w(t-1)}$	Author's calculation using $w(t)$ variable
$L(t)$	Loans to individuals, billions of rubles	CBR

$vL(t)$	Loans to individuals, USD bn	CBR
$S(t)$	Individual deposits, billions of rubles	CBR
$vS(t)$	Individual deposits, USD bn	CBR
$M(t)$	Current accounts of individuals, billions of rubles	CBR
$K(t)$	Loans granted to individuals, billions of rubles	CBR
$vK(t)$	Loans granted to individuals, billions of dollars	CBR
$V(t)$	Deposits received from individuals, billions of rubles	CBR
$vV(t)$	Deposits received from individuals, billions of dollars	CBR
$r_l(t)$	The interest rate on loans to individuals in rubles	CBR
$r_{vl}(t)$	The interest rate on loans to individuals in US dollars	CBR
$r_s(t)$	The interest rate on deposits of individuals in rubles	CBR
$r_{vs}(t)$	The interest rate on deposits of individuals US dollars	CBR
$\beta_l(t)$	Frequency of repayment of loans to individuals (rubles)	CBR
$\beta_{vl}(t)$	Frequency of repayment of loans to individuals (dollars)	CBR
$\beta_s(t)$	Frequency of return of deposits of individuals (rubles)	CBR
$\beta_{vs}(t)$	Frequency of return of deposits of individuals (dollars)	CBR
$ovQ(t)$	Currency purchase costs, USD billions	CBR
$vQ(t)$	Consumer foreign currency, USD billions	CBR
$\beta_{vq}(t)$	Fixed parameter (data NA)	Constant 0.1
$oB(t)$	Real estate purchase expenses	FSSS
$B(t)$	Real estate amount, billions of rubles	FSSS (balance of income and expenses, obdx)
$\beta_b(t)$	Fixed parameter (data NA)	Constant 0.001
$r_b(t)$	Real estate profitability rate	FSSS
$OC(t)$	$M(t)$ minus all sums on the consumer balance	Author's calculation

Figures below (data – blue line, model estimation – orange line) describe the result of equations (2.13) - (2.27), which provide model estimation results.

The equation (2.14) describes endogenous labor in mil people:

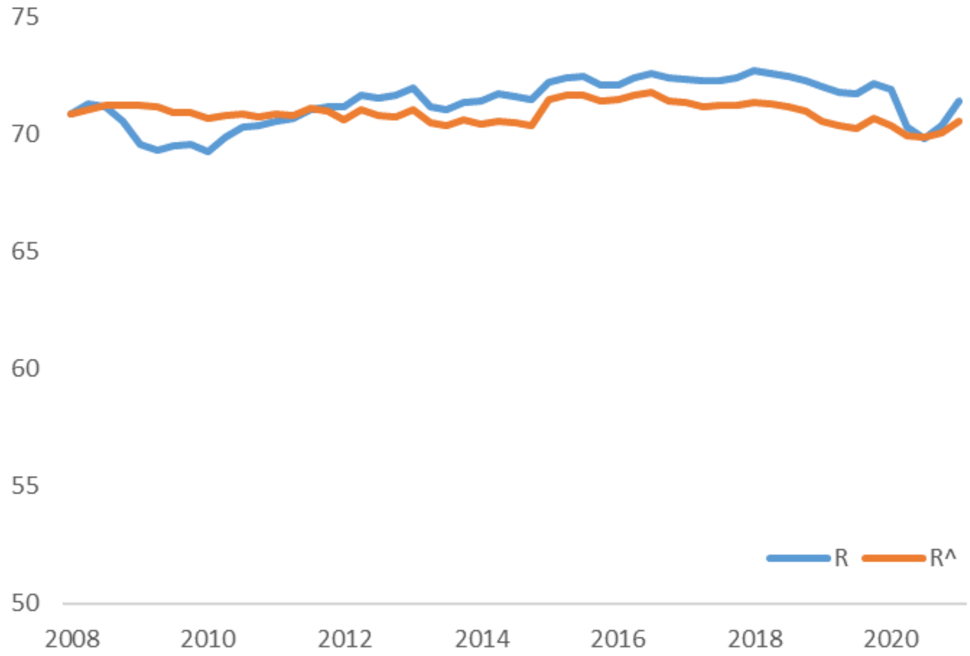

Fig. 3.1. Labor $R(t)$, mil people

The equation (2.13) results in consumer consumption:

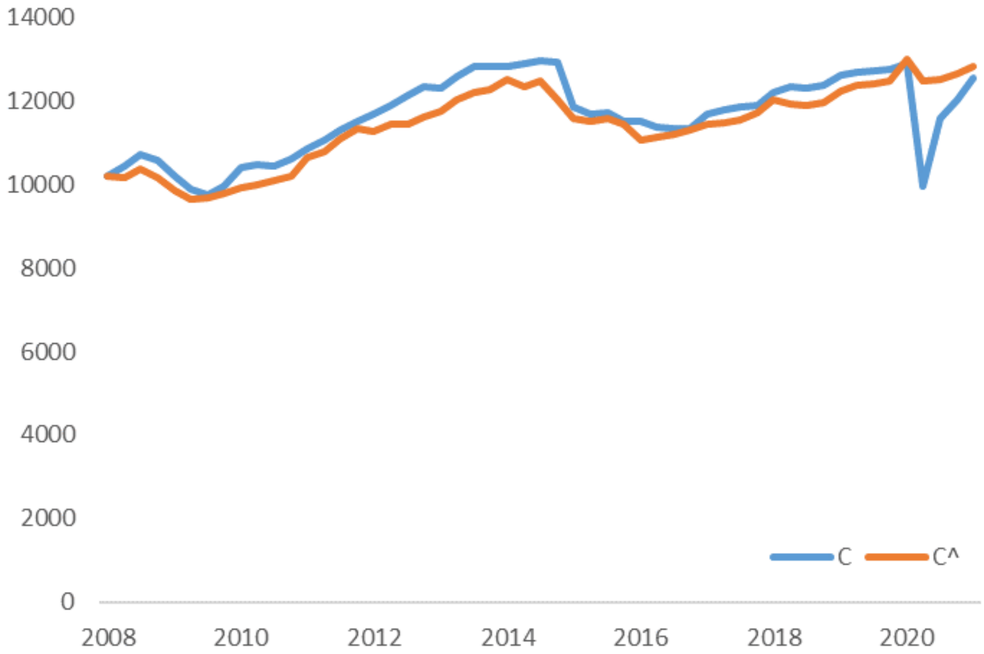

Fig. 3.2. Consumption $C(t)$, billions of rubles

The equation (2.15) describes consumer loans in national currency:

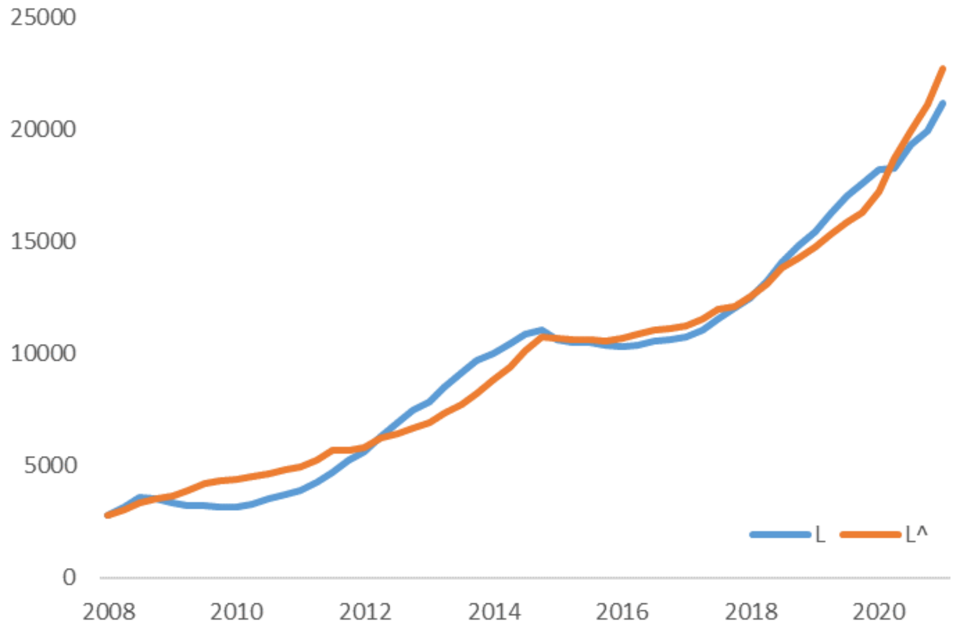

Fig. 3.3. Loans in national currency $L(t)$, billions of rubles

The equation (2.16) stays for consumer loans in foreign currency:

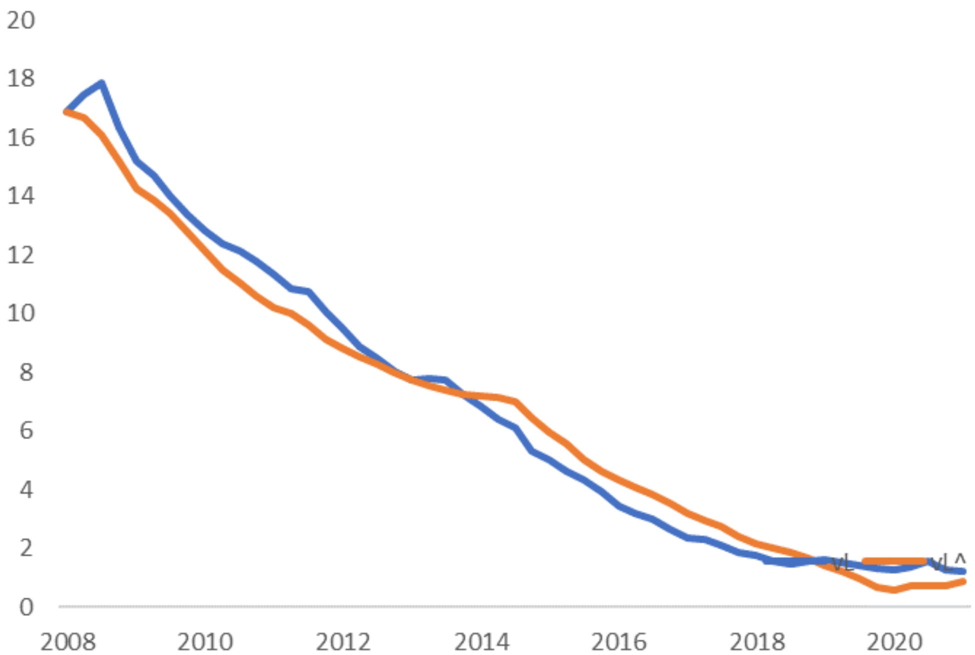

Fig. 3.4. Loans in foreign currency $vL(t)$, USD billion

The equation (2.17) describes consumer deposits in national currency:

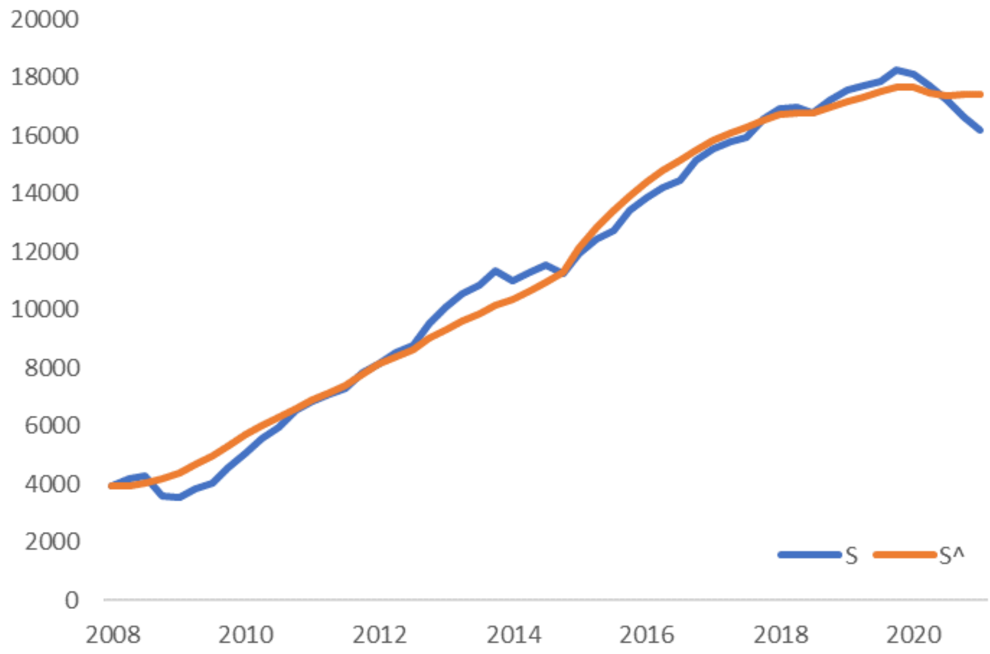

Fig. 3.5. Deposits in national currency $S(t)$, billions of rubles

The equation (2.18) results in consumer deposits in foreign currency:

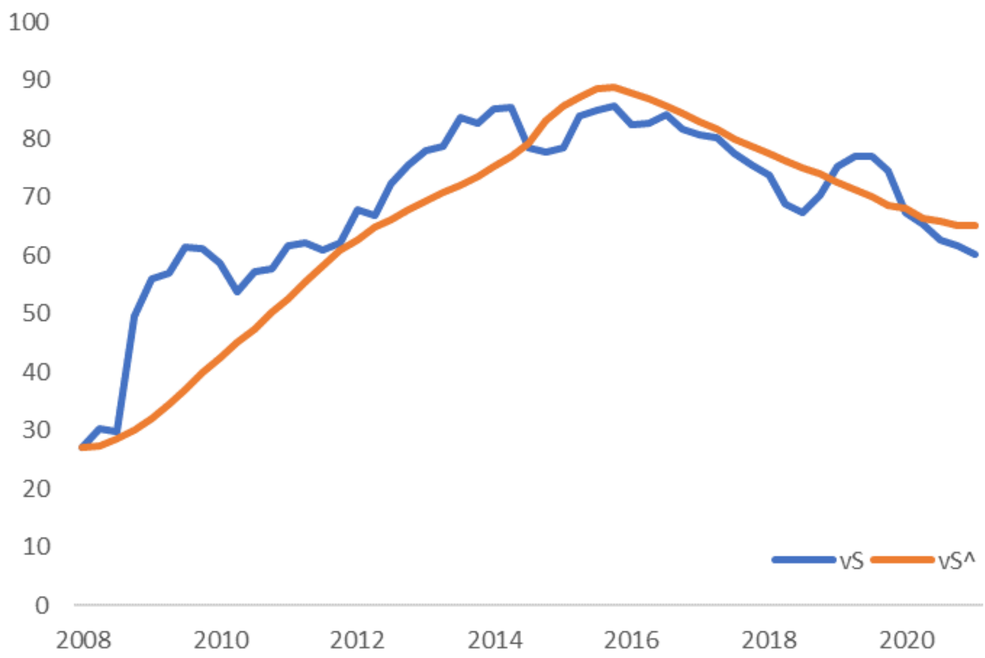

Fig. 3.6. Deposits in foreign currency $vS(t)$, USD billions

The equation (2.20) describes consumer foreign currency:

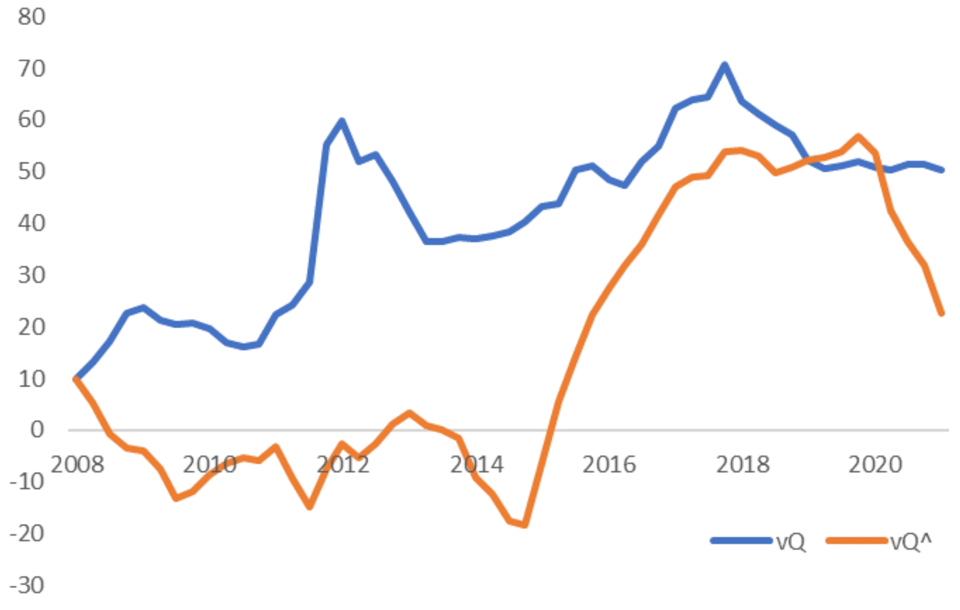

Fig. 3.7. Foreign currency $vQ(t)$, USD billions

The equation (2.19) describes consumer real estate amount:

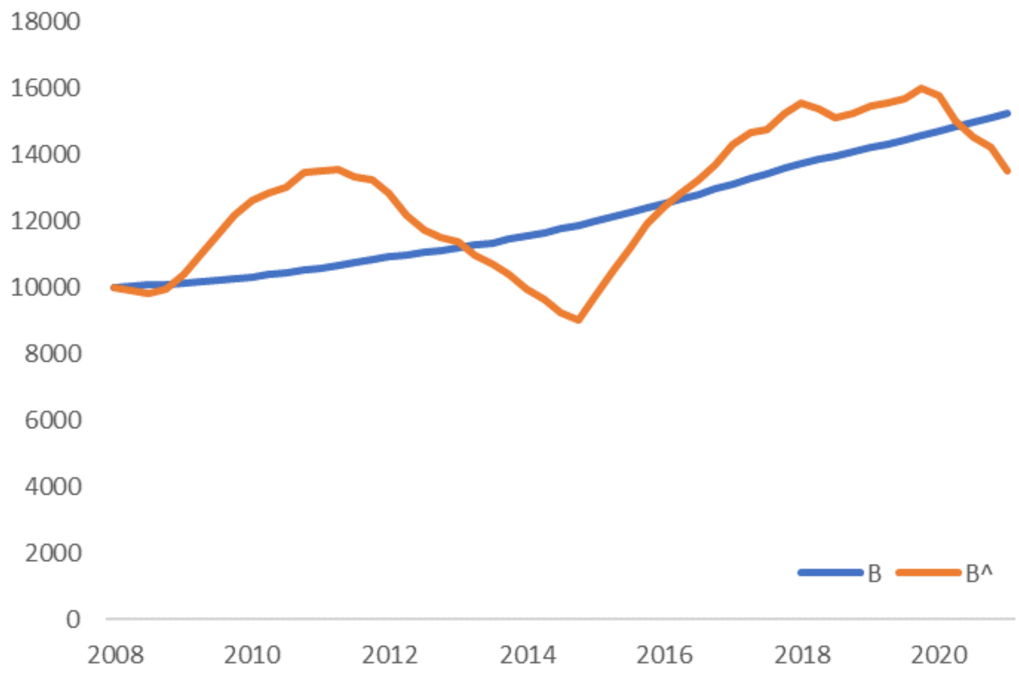

Fig. 3.8. Real estate amount, billions of rubles

The equation (2.7) describes consumer financial balance:

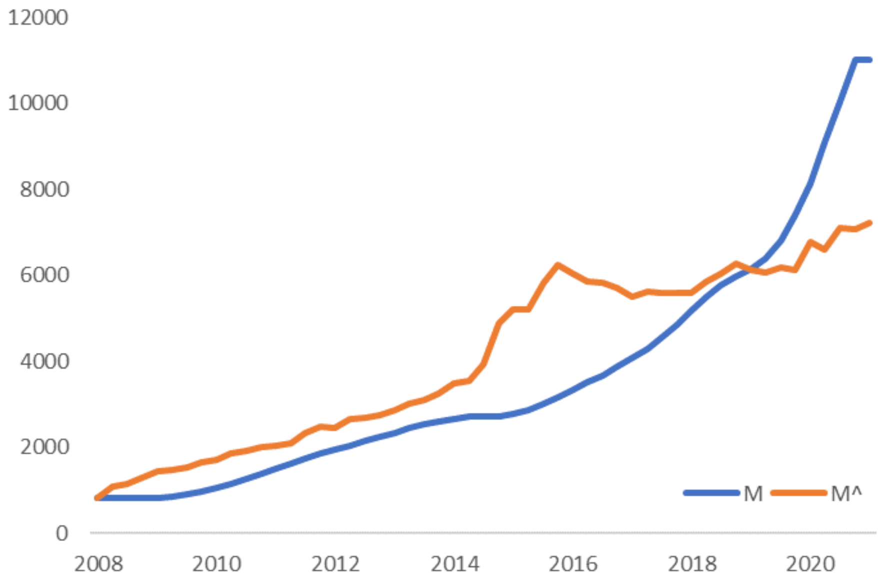

Fig. 3.9. Consumer current assets, billions of rubles

The figures above demonstrate high accuracy for consumption $C(t)$, labor $R(t)$, loans $L(t), vL(t)$, and deposits $S(t), vS(t)$. However, real estate $B(t)$ and currency $Q(t)$ description accuracy are low, resulting in lower accuracy for consumer current accounts $M(t)$ description. Nevertheless, the figures represent overall trends. The reason for observed performance is that it is clear how to describe loans and deposits (see [7]). The currency and real estate variables description is studied briefly in the given type of problems. It is one of the research topics for future studies. Table 3.2 shows the accuracy metric MAPE for two periods: including and excluding pandemic (Q1-2020 – Q1-2021). Overall model accuracy is significantly better when the pandemic period is excluded. It is necessary to point out that model performance facing unpredictable events as pandemic crisis remains good enough.

Table 3.2. Model accuracy measures (MAPE)

Model Estimated Variable	MAPE (Q1-2008 to Q1-2021)	MAPE (Q1-2008 to Q4-2019)
Labor $R(t)$	1.25%	1.13%
Consumption $C(t)$	3.46%	1.18%
Amount of currency $vQ(t)$	77.93%	89.4%
Amount of real estate $B(t)$	11.22%	12.16%
Loans in national currency $L(t)$	10.14%	11.34%
Loans in foreign currency $vL(t)$	17.47%	11.61%
Deposits in national currency $S(t)$	5.26%	4.16%
Deposits in foreign currency $vS(t)$	10.62%	8.61%
Consumer current accounts $M(t)$	39.69%	18.39%

It is necessary to address the Lagrange multiplier method to clarify parameters a_i, b_i, c_i in equations (2.12), (2.21)-(2.26). The Lagrange multiplier method results in a system of inequalities for dual variables and corresponding control variables where one variable becomes equal to zero and the other one to non-zero during the quarterly interval. The system stays in one of many possible regime states. Variables change regimes frequently, which can be interpreted as when consumers take loans and buy a foreign currency. In other moments he makes deposits or everything altogether. The consumer consistently changes the regimes by performing various operations. Data is discrete, and the system state is unobservable in statistics during the quarter, but this approach helps restore this information. Consumer makes his decision about the regime with the help of exchange and interest rates included in $\rho(t)$ (consumer profitability rate). Parameters a_i and b_i describe how long the system stays in one regime during quarter intervals based on profitability (rates) of available financial instruments to the consumer. c_i reflects all other money flows not connected with financial instruments (consumption, wage, transfers, and others) and helps identify regime states when the consumer does not operate with financial instruments, but performs other activities, e.g., consuming goods and services. To sum up, parameters a_i, b_i, c_i are elasticity frequency ratios to income variables. All above summarizes the idea of relaxation of complementary slackness conditions.

As it was stated, not all variables are described with high accuracy since it is still a research question of how to describe foreign currency amounts available to the consumer. That is why the received system contains a noticeable disturbance variable $vQ(t)$ that influences overall model performance.

4. CONCLUSION

The method of relaxation of complementary slackness conditions allows including various financial instruments in the model of an aggregated consumer. It helps to describe consumer financial balances in detail corresponding to available data. On the example of the Russian economy, it is proved that this method can provide high modeling accuracy even in crisis periods, but a study of foreign currency amounts available to the consumer is still needed.

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