Improved Airline Seat Inventory Control Policies under Parametric Uncertainty of Customer Demand Models

Nicholas A. Nechval[1], Konstantin N. Nechval[2] and Maris Purgailis[1]
[1] *University of Latvia, EVF Research Institute, Statistics Department, Raina Blvd 19, LV-1050 Riga, Latvia*
[2] *Transport and Telecommunication Institute, Applied Mathematics Department, Lomonosov Street 1, LV-1019 Riga, Latvia*

Abstract
Most models, which are used for solving airline seat inventory control problems, are developed in the literature under the assumptions that the parameter values of the models are known with certainty. When these models are applied to solve real-world problems, the parameters are estimated and then treated as if they were the true values. The risk associated with using estimates rather than the true parameters is called estimation risk and is often ignored. When data are limited and/or unreliable, estimation risk may be significant, and failure to incorporate it into the model design may lead to serious errors. In this paper, we consider the static and dynamic problems of airline seat inventory control under parametric uncertainty, which are invariant with respect to a certain group of transformations. Since common practice for airlines is to charge several different fares for a common pool of seats, this paper presents the policies that have been used to address the problem of when to refuse booking requests for a given fare level to save the seat for a potential request at a higher fare level. In this paper, we present the innovative technologies for constructing the static and dynamic policies of the airline seat inventory control.on the basis of the 'unbiasedness performance index'. The idea of prediction of a future cumulative customer demand for the seats on a flight via the order statistics from the underlying distribution, introduced in the paper, allows one to use the invariant embedding technique in order to eliminate the unknown parameters from the problem and to use the previous and current sample data as completely as possible. The proposed unbiased static and dynamic policies are more efficient as compared with the policies, where the unknown parameters of the airline customer demand models are estimated and then treated as if they were the true values. An illustrative example is given.

Keywords Airlines, demand, uncertainty, airline booking, optimization

1 Introduction

Passenger reservations systems have evolved from low level inventory control processes to major strategic information systems. Today, airlines and other transportation companies view revenue management systems and related information technologies as critical determinants of future success. Indeed, expectations of revenue gains that are possible with expanded revenue management capabilities are now driving the acquisition of new information technology. Each advance in information technology creates an opportunity for more comprehensive reservations control and greater integration with other important transportation planning functions.

The airline seat inventory control problem lies at the heart of airline revenue management. It is common practice for airlines to sell a pool of identical seats at different prices according to different booking classes to improve revenues in a very competitive market. In other words, airlines sell the same seat at different prices according to different types of travelers (first class, business and economy) and other conditions. The question then arises whether to offer seats at a relatively low price at a given time with a given number of seats remaining or to wait for the possible arrival of a higher paying customer. Assigning seats in the same compartment to different fare classes of passengers in order to improve revenues is a major problem of airline seat inventory control. This problem has been considered in numerous papers. For details, the reader is referred to a review of yield management, as well as perishable asset revenue management, by Weatherford et al. [1], and a review of relevant mathematical models by Belobaba [2].

This paper deals with the airline seat inventory control problem when customers for different fare levels are booked into a common seating pool in the aircraft. The following assumptions are made: (1) single-leg flight: bookings are made on the basis of a single departure and landing; no allowance is made for the possibility that bookings may be part of larger trip itineraries, (2) independent demands: the demands for different fare classes are stochastically independent, (3) low before high demands: the lowest fare reservations requests arrive first, followed by the next lowest, etc., (4) no cancellations: cancellations, no-shows and overbooking are not considered, (5) nested classes: any fare class can be booked into seats not taken by bookings in lower fare classes, (6) fare classes: the business and economy fare classes are considered.

The first purpose of this paper is to present the innovative information technologies for constructing the static and dynamic policies of the airline seat inventory control on the basis of the 'unbiasedness performance index'. The static and dynamic policies (unbiased) are more efficient (from the point of view of airline revenue management) as compared with the policies, where the unknown parameters of the airline customer demand models are estimated and then treated as if they were the true values.

The second purpose of this paper is to introduce the idea of prediction of a future cumulative customer demand for the seats on a flight via the order statistics from the underlying distribution, where only the functional form of the distribution is specified, but some or all of its parameters are unspecified. This idea allows one to use the technique of invariant embedding of sample statistics in a performance index in order to eliminate the unknown parameters from the problem [3-4]. The technique represents a simple and computationally attractive statistical method based on the constructive use of the invariance principle in mathematical statistics. Unlike the Bayesian approach, an invariant embedding technique is independent of the choice of priors, i.e., subjectivity of investigator is eliminated from the problem. It allows one to find the improved invariant statistical decision rules, which have smaller risk than any of the well-known traditional statistical decision rules, and to use the previous and current sample data as completely as possible.

2 State-of-the-art and Progress Beyond

Airline seat inventory control is a very profitable tool in the airline industry. A major problem of airline seat inventory control is to sell the same seat at different prices according to different types of travelers (first class, business and economy) and other conditions in order to improve revenues. This problem has been considered in numerous papers. Littlewood [5] was the first to propose a solution method of the seat inventory control problem for a single leg flight with two fare classes. The idea of his scheme is to equate the marginal revenues in each of the two fare classes. He suggests closing down the low fare class when the certain revenue from selling low fare seat is exceeded by the expected revenue of selling the same seat at the higher fare. That is, low fare booking requests should be accepted as long as

$$c_2 \geq c_1 Pr\{Y_1 > \mu_1\}, \quad (1)$$

where c_1 and c_2 are the high and low fare levels respectively, Y_1 denotes the demand for the high fare (or business) class,μ_1 is the number of seats to protect for the high fare class and $Pr\{Y_1 > \mu_1\}$ is the probability of selling more than μ_1 protected seats to high fare class customers. The smallest value of μ_1 that satisfies the above condition is the number of seats to protect for the high fare class, and is known as the protection level of the high fare class customers. The concept of determining a protection level for the high fare class can also be seen as setting a booking limit, a maximum number of bookings, for the low fare class. Both concepts restrict the number of bookings for the low fare class in order to accept bookings for the high fare class.

It should be remarked that there is no protection level for the low fare (or economy) class;μ_2 is the booking limit, or number of seats available, for the low fare class; the low fare class is open as long as the number of bookings in this class remains less than this limit. Thus,is the booking limit, or number of seats available, for the low fare class; the low fare class is open as long as the number of bookings in this class remains less than this limit. Thus, $\mu_1+\mu_2$ is the booking limit or number of seats available, for the high fare class at time. The high fare class is open as long as the number of bookings in this and low classes remain less than this limit.

Richter [6] gave a marginal analysis, which proved that (1) gives an optimal allocation (assuming certain continuity conditions). Optimal policies for more than two classes have been presented independently by Curry [7], Wollmer [8], and Brumelle & McGill [9].

3 Airline Booking Policies which are Used in Practice

3.1 Static Airline Booking Policy under Complete Information

It will be noted that (1) represents the static policy of airline seat inventory control (or airline booking) under complete information. If F_θ , the probability distribution function of Y_1 with the parameter θ (in general, vector), is continuous and strictly increasing, the definition (1) of μ_1 is equivalent to

$$\mu_1 = \arg\left(\bar{F}_\theta(\mu_1) = \gamma\right) \tag{2}$$

where

$$\gamma = c_1/c_2, \tag{3}$$

$$\bar{F}_\theta(\tau_j) = 1 - F_\theta(\tau_j). \tag{4}$$

3.2 Static Airline Booking Policy under Parametric Uncertainty

In practice, under parametric uncertainty, i.e. when the parameter θ is unknown, the performance index,

$$\bar{F}_\theta(\mu_1) = \gamma, \tag{5}$$

is usually used to construct the static policy given by

$$\mu_1 = \arg\left(\bar{F}_{\hat{\theta}}(\mu_1) = \gamma\right), \tag{6}$$

where $\hat{\theta}$ represents the maximum likelihood estimator of θ . The performance index (5) is named as 'maximum likelihood performance index'. The static policy (6) based on (5) is named as *'static maximum likelihood airline booking policy'.*

3.3 Dynamic Airline Booking Policy under Parametric Uncertainty

The static policy of airline booking is optimal as long as no change in the probability distributions of the customer demand is foreseen. However, information on the actual customer demand process can reduce the uncertainty associated with the estimates of demand. Hence, repetitive use of a static policy over the booking period, based on the most recent demand and capacity information, is the general way to proceed.

4 Improved Airline Booking Policies Proposed in the Paper

4.1 Static Unbiased Airline Booking Policy under Parametric Uncertainty

This policy is based on the performance index,

$$E_\theta\{\bar{F}_\theta(\mu_1)\} = \gamma, \tag{7}$$

which takes into account (2) and the previous data of cumulative customer demand Y_1 for the seats on a flight. It allows one to construct the static airline booking policy given by

$$\mu_1^{(\mu nb)} = \arg\left(E_\theta\{\bar{F}_\theta(\mu_1)\} = \gamma\right), \tag{8}$$

where $\mu_1 \equiv \mu_1(\hat{\theta})$,$\hat{\theta}$ represents either the maximum likelihood estimator of θ or sufficient statistic S for θ, i.e., $\mu_1 \equiv \mu_1(S)$ The performance index (7) is named as 'unbiasedness performance index'.The static policy (8), which is based on (7), is named as *'static unbiased airline booking policy'*.

The relative bias of the static airline booking policy is given by

$$\gamma(\mu_1) = \frac{\left|E_\theta\{\bar{F}_\theta(\mu_1)\} - \gamma\right|}{\gamma} 100\% \tag{9}$$

4.2 Dynamic Airline Booking Policy under Complete Information

In this section, we consider a flight for a single departure date with m predefined reading dates at which the dynamic policy is to be updated, i.e., the booking period before departure is divided into m readings periods:$(0, \tau_1], (\tau_1, \tau_2], \ldots, (\tau_{m-1}, \tau_m]$ determined by the m reading dates:$\tau_1, \tau_2, \ldots, \tau_m$. These reading dates are indexed in increasing order:$0 < \tau_1 < \tau_2 < \ldots < \tau_m$, where $(\tau_{m-1}, \tau_m]$ denotes the reading period immediately preceding departure, and $\tau_m]$ is at departure. Typically, the reading periods that are closer to departure cover much shorter periods of time than those further from departure. For example, the reading period immediately preceding departure may cover 1 day whereas the reading period 1-month from departure may cover 1 week.

Let us suppose that the cumulative passenger demand for the high fare class at the kth reading date ($time\ \tau_k, 1 \leq k \leq m$) is Y_{1k} representing the kth order statistic from the underlying distribution with the probability distribution function $G_\theta(y_{1k})$, where θ is a parameter (in general, vector). In other words,Y_{1k} represents the number of seats sold for the customers of the high fare class at the kth reading date. We assume that the cumulative passenger demands for the high and low fare classes are stochastically independent. Each booking of a seat of the high fare class generates average revenue of c_1. Each booking of a seat of the low fare class generates average revenue of c_2 , where $c_2 < c_1$.Let μ_{1k} be an individual protection level for the high fare class at time τ_k (the kth

reading date).This many seats are protected for the high fare class from the low fare class. There is no protection level for the low fare class;μ_{2k} is the booking limit for the low fare class at time τ_k; the low fare class is open as long as the number of bookings in this class remains less than this limit. Thus,$\mu_{1k} + \mu_{2k}$ is the booking limit for the high fare class at time τ_k The high fare class is open as long as the number of bookings in this and low classes remain less than this limit. The maximum number of seats that may be booked by fare classes in the next at time τ_k prior to flight departure is the number of unsold seats μ_k°.

Under the complete information, the dynamic airline booking policy is given by

$$\mu_{1k} = \arg\left(\bar{G}_\theta(\mu_{1k}|y_{1k}) = \gamma\right), \quad k = 1, 2, \ldots, m-1, \tag{10}$$

where

$$\bar{G}_\theta(\mu_{1k}|y_{1k}) = 1 - G_\theta(\mu_{1k}|y_{1k}), \tag{11}$$

$G_\theta(\mu_{1k}|y_{1k})$ represents the conditional probability distribution function of the mth order statistic Y_{1m} . The number of unsold seats protected for the high fare class from the low fare class in the next at time τ_k prior to flight departure is the number of unsold seats,μ_{1k}°,which is given by

$$\mu_{1k}^\circ = min(\mu_k^\circ, \mu_{1k} - y_{1k}). \tag{12}$$

4.3 Dynamic Unbiased Airline Booking Policy under Parametric Uncertainty

Under the parametric uncertainty, the dynamic unbiased airline booking policy is given by

$$\mu_{1k}^{unb} = \arg\left(E_\theta\{\bar{G}_\theta(\mu_{1k}|y_{1k})\} = \gamma\right), \quad k = 1, 2, \ldots, m-1, \tag{13}$$

where $\mu_{1k} \equiv \mu_{1k}(\hat{\theta})$,$\hat{\theta}$ represents either the maximum likelihood estimator of θ or sufficient statistic S for θ, i.e.,$\mu_{1k} \equiv \mu_{1k}(S)$.The number of unsold seats protected for the high fare class from the low fare class in the next at time τ_k prior to flight departure is the number of unsold seats μ_{1k}°,which is given by

$$\mu_{1k}^{\circ(unb)} = min(\mu_k^\circ, \mu_{1k}^{unb} - y_{1k}). \tag{14}$$

5 Mathematical Preliminaries

Theorem 1 *Let $X_1 \leq \ldots \leq X_k$ be the first k ordered observations (order statistics) in a sample of size m from a continuous distribution with some probability density function $f_\theta(x)$ and distribution function $F_\theta(x)$ where θ is a parameter (in general, vector). Then the joint probability density function of $X_1 \leq \ldots \leq X_k$ and the lth order statistics $X_l (1 \leq k \leq l \leq m)$ is given by*

$$g_\theta(x_1, \ldots, x_k, x_l) = g_\theta(x_1, \ldots, x_k) g_\theta(x_l|x_k), \tag{15}$$

where

$$g_\theta(x_1,\ldots,x_k) = \frac{m!}{(m-k)!}\prod_{i=1}^{k} f_\theta(x_i)\Big[1-F_\theta(x_k)\Big]^{m-k}, \tag{16}$$

$$g_\theta(x_l|x_k) = \frac{(m-k)!}{(l-k-1)!(m-l)!}\Big[\frac{F_\theta(x_l)-F_\theta(x_k)}{1-F_\theta(x_k)}\Big]^{l-k-1}\Big[1-\frac{F_\theta(x_l)-F_\theta(x_k)}{1-F_\theta(x_k)}\Big]^{m-l}$$

$$\frac{f_\theta(x_l)}{1-F_\theta(x_k)} = \frac{(m-k)!}{(l-k-1)!(m-l)!}\sum_{j=0}^{k-l-1}\left(\begin{array}{c} l-k-1 \\ j \end{array}\right)(-1)^j\Big[\frac{1-F_\theta(x_l)}{1-F_\theta(x_k)}\Big]^{m-l+j} \tag{17}$$

$$\frac{f_\theta(x_l)}{1-F_\theta(x_k)} = \frac{(m-k)!}{(l-k-1)!(m-l)!}\sum_{j=0}^{m-l}\left(\begin{array}{c} m-l \\ j \end{array}\right)(-1)^j\Big[\frac{F_\theta(x_l)-F_\theta(x_k)}{1-F_\theta(x_k)}\Big]^{l-k-1+j}$$

$$\frac{f_\theta(x_l)}{1-F_\theta(x_k)}$$

represents the conditional probability density function of X_l *given* $X_k = x_k$.

Proof. The joint density of $X_1 \leq \ldots \leq X_k$ and X_l is given by

$$\begin{aligned} & g_\theta(x_1,\ldots,x_k,x_l) \\ =& \frac{m!}{(l-k-1)!(m-l)!}\prod_{i=1}^{k} f_\theta(x_i)\Big[F_\theta(x_l)-F_\theta(x_k)\Big]^{l-k-1} f_\theta(x_l)\Big[1-f_\theta(x_l)\Big]^{m-l} \\ =& g_\theta(x_1,\ldots,x_k)g_\theta(x_l|x_k). \end{aligned} \tag{18}$$

It follows from (4) that

$$g_\theta(x_l|x_1,\ldots,x_k) = \frac{g_\theta(x_1,\ldots,x_k,x_l)}{g_\theta(x_1,\ldots,x_k)} = g_\theta(x_l|x_k), \tag{19}$$

i.e., the conditional distribution of X_l, given $X_i = x_i$ for all $i = 1,\ldots,k$, is the same as the conditional distribution of X_l, given only $X_k = x_k$, which is given by (17). This ends the proof.

Corollary 1.1. The conditional probability distribution function of X_l given $X_k = x_k$ is

$$\begin{aligned} & P_\theta\{X_l \leq x_l|X_k = x_k\} \\ =& 1-\frac{(m-k)!}{(l-k-1)!(m-l)!}\sum_{j=0}^{l-k-1}\left(\begin{array}{c} l-k-1 \\ j \end{array}\right)\frac{(-1)^j}{m-l+1+j}\Big[\frac{1-F_\theta(x_l)}{1-F_\theta(x_k)}\Big]^{m-l+1+j} \\ =& \frac{(m-k)!}{(l-k-1)!(m-l)!}\sum_{j=0}^{m-k}\left(\begin{array}{c} m-l \\ j \end{array}\right)\Big[\frac{F_\theta(x_l)-F_\theta(x_k)}{1-F_\theta(x_k)}\Big]^{l-k+j}. \end{aligned} \tag{20}$$

Corollary 1.2. Let $X_1 \leq \ldots \leq X_k$ be the first k order statistics in a sample of size m from the two-parameter Weibull distribution with the probability density function

$$f_\theta(x) = \frac{\delta}{\beta}\Big(\frac{x}{\beta}\Big)^{\delta-1} \exp\Big[-(\frac{x}{\beta})^{\delta}\Big](x > 0), \tag{21}$$

where $\theta = (\beta, \delta)$, $\beta > 0$ and $\delta > 0$ are the scale and shape parameters, respectively. Then the conditional probability distribution function of X_l given $X_k = x_k$ is

$$P_\theta\{X_l \leq x_l | X_k = x_k\} = 1 - \frac{(m-k)!}{(l-k-1)!(m-l)!}\sum_{j=0}^{l-k-1}\binom{l-k-1}{j} \frac{(-1)^j}{m-l+1+j}\Big[\exp(-\frac{x_l^\delta - x_k^\delta}{\beta^\delta})\Big]^{m-l+1+j}. \tag{22}$$

Theorem 2 *If in (22) the scale parameter β is unknown, then the predictive probability distribution function of X_l based on (x_k, δ) is given by*

$$P_\delta\Big\{\Big(\frac{X_l}{X_k}\Big)^\delta \leq \Big(\frac{x_l}{x_k}\Big)^\delta)\Big\} = 1 - \frac{m!}{(l-k-1)!(m-l)!} \times \sum_{j=0}^{l-k-1}\binom{l-k-1}{j} \frac{(-1)^j}{m-l+1+j}\Big(\Pi_{s=0}^{k-1}\Big[\Big(\Big(\frac{x_l}{x_k}\Big)^\delta - 1\Big)(m-j+1+j) + (m-k+1+s)\Big]\Big)^{-1}. \tag{23}$$

Proof. We reduce (22) to

$$P_\theta\Big\{\Big(\frac{X_l}{X_k}\Big)^\delta \leq \Big(\frac{x_l}{x_k}\Big)^\delta | \Big(\frac{X_k}{\beta}\Big)^\delta = \Big(\frac{x_k}{\beta}\Big)^\delta\Big\} = 1 - \frac{(m-k)!}{(l-k-1)!(m-l)!} \sum_{j=0}^{l-k-1}\binom{l-k-1}{j} \times \frac{(-1)^j}{m-l+1+j}\Big[\exp\Big(-\omega[\nu^\delta - 1]\Big)\Big]^{m-l+1+j} \tag{24}$$
$$= P_\delta\{V^\delta \leq \nu^\delta | W = \omega\}$$

where $V = X_l/X_k$ is the ancillary statistic whose distribution does not depend on the parameter β. Since X_k does not depend on V,$W = (X_k/\beta)^\delta$ is the pivotal quantity, whose distribution is known and does not depend on the parameters β and δ, we eliminate the parameter β from the problem as

$$P_\delta\{X_l \leq x_l\} = \int_0^\infty P_\theta\{X_l \leq x_l | X_k = x_k\} g_\theta(x_k) dx_k, \tag{25}$$

where

$$g_\theta(x_k) = \frac{m!}{(k-1)!(m-k)!} F_\theta^{k-1}(x_k)\Big[1 - F_\theta(x_k)\Big]^{m-k} f_\theta(x_k),\ x_k \in (0, \infty), \tag{26}$$

represents the probability density function of the kth order statistic X_k.Indeed, it follows from (26) that

$$\begin{aligned} g_\theta(x_k)dx_k &= \frac{m!}{(k-1)!(m-k)!}\Big[1-\exp\Big(-(\frac{x_k}{\beta})^\delta\Big)\Big]^{k-1}\exp\Big(-(\frac{x_k}{\beta})^{\delta(m-k)}\Big) \\ &\exp\Big(-(\frac{x}{\beta})^\delta\Big)d(\frac{x}{\beta})^\delta \\ &= \frac{m!}{(k-1)!(m-k)!}\big[1-e^{-\omega}\big]^{k-1}e^{-\omega(m-k+1)}d\omega \\ &= g(\omega)d\omega. \end{aligned} \tag{27}$$

It follows from (24) and (27) that

$$\begin{aligned} &P_\delta\{V^\delta \le \nu^\delta\} \\ =&\int_0^\infty P_\delta\{V^\delta \le \nu^\delta | W=\omega\}g(\omega)d(\omega) \\ =&1-\frac{m!}{(l-k-1)!(m-l)!}\sum_{j=0}^{l-k-1}\binom{l-k-1}{j}\frac{(-1)^j}{m-l+1+j} \\ &\Big(\Pi_{s=0}^{k-1}\Big[(\nu^\delta-1)(m-l+1+j)+(m-k+1+s)\Big]\Big)^{-1}. \end{aligned} \tag{28}$$

Now (23) follows from (28). This ends the proof.

Corollary 2.1. If the parameter $\delta = 1$, i.e., we deal with the exponential distribution,

then the predictive probability distribution function of X_l based on x_k is given by

$$\begin{aligned} &P\Big\{\Big(\frac{X_l}{X_k}\le x_l x_k\Big)\Big\} \\ =&1-\frac{m!}{(l-k-1)!(m-l)!}\times\sum_{j=0}^{l-k-1}\binom{l-k-1}{j}\frac{(-1)^j}{m-l+1+j} \\ &\Big(\Pi_{s=0}^{k-1}\Big[(\frac{x_l}{x_k-1})(m-l+1+j)+(m-k+1+s)\Big]\Big)^{-1}. \end{aligned} \tag{29}$$

6 Illustrative Example of Airline Booking Policies

Let $X_1,\ldots,X_n$ be the random sample of the previous independent observations of the cumulative customer demand for the high fare class, which follow the exponential distribution with the probability density function (21)($\delta = 1$) , where the parameter β is unknown. Then the static policies of airline booking under parametric uncertainty are given as follows.

The static maximum likelihood airline booking policy follows from (6):

$$\mu_1^{ml} = \ln \gamma^{-S/n}, \tag{30}$$

where $S = \sum_{i=1}^{n} X_i$ is the sufficient statistic for β,with

$$V = S/\beta \sim f(\nu) = \frac{1}{\Gamma(n)} \nu^{n-1} \exp(-\nu),\ \nu \geq 0, \tag{31}$$

and the relative bias,

$$r(\mu_1^{ml}) = \frac{\left|E_\theta\{\bar{F}_\theta(\mu_1^{ml})\} - \gamma\right|}{\gamma} 100\% = 1 + \frac{(1 + \ln \gamma^{-1/n})^{-1} - \gamma}{\gamma} 100\%. \tag{32}$$

If, say,$n = 1$ and $\gamma = 0.4$, then $r_{rb}(\mu_1^{ml}) = 30\%$.Thus, in this example the static maximum likelihood airline booking policy has the relative bias equal to 30%.It follows that the protection level for customers of the high fare class will be determined incorrectly. This may lead to serious loss.

*The static unbiased airline booking policy*follows from (8):

$$\mu_1^{unb} = [\gamma^{-1/n} - 1]S, \tag{33}$$

where the relative bias $r(\mu_1^{unb}) = 0$.

*The dynamic unbiased airline booking policy*follows from (13) and (29):

$$\begin{aligned} \mu_{1k}^{unb} = \arg \Bigg(\frac{m!}{(m-k-1)!} \sum_{j=0}^{m-k-1} \binom{m-k-1}{j} \frac{(-1)^j}{1+j} \Big(\Pi_{s=0}^{k-1} \Big[\big(\frac{\mu_{1k}}{y_{1k}} - 1\big)(1+j) \\ + (m-k+1+s) \Big] \Big)^{-1} = \gamma \Bigg), \quad k = 1, 2 \ldots, m-1, \end{aligned} \tag{34}$$

$$\mu_{1k}^{\circ(unb)} = min\Big(\mu_k^{\circ}, \mu_{1k}^{unb}, -y_{1k}\Big). \tag{35}$$

7 Conclusion

The methodology, which is developed in this paper for the use in the airline industry under parametric uncertainty of airline customer demand models, may be found to be useful in other industries such as hotels, car rental companies, shipping companies, etc. While the details of problems considered in this project can change significantly from one industry to the next, the focus is always on making better demand decisions and not manually with guess work and intuition but rather scientifically with models and technology, all implemented with disciplined processes and systems.

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Corresponding author

Nicholas Nechval can be contacted at:jnechval@junik.lv