Adv Syst Sci Appl 2022; **03**; 18-35
Published online at https://ijassa.ipu.ru.

Models of Industrial Risk Control Systems

Mikhail Geraskin[1], Elena Rostova[2*]

[1)] *Samara National Research University, Samara, Russia*
E-mail: innovation@ssau.ru
[2)] *Samara National Research University, Samara, Russia*
E-mail: el_rostova@mail.ru

Abstract: We investigate a problem of searching for Pareto equilibrium sets of an insurance rate and an industrial damage utilization price. We consider a system, which, in the case of industrial accidents, arises around an industrial firm. An industrial firm, a waste utilization firm, and an insurance company are considered as the system's agents. We develop profit functions for the agents, and we determine compromise prices on waste utilization and insurance, which provide the system's stability. We analyze the set of industrial risk control systems with a various number of the agents and the agent's relations. A problem of determining an optimal solution is solved on the basis of maximizing agents' profit functions. The sets of an equilibrium industrial damage utilization price and an equilibrium insurance rate are defined as Pareto equilibrium. A problem of determining the set of an insurance rate is solved taking into account constraints according to requirements of an industrial firm and an insurance company. A problem of determining the set of an industrial damage utilization price is solved taking into account constraints according to requirements of an industrial firm and a waste utilization firm. We consider the following models of industrial risk control systems: agents have a strong relation and a weak relation, additionally, one agent of each type and of many agents of the same type.

Keywords: industrial risk, insurance, waste utilization, optimization, risk control

1. INTRODUCTION

The industrial risk control is an important problem for every firm because the influence of different external market factors. The risk control problems cover a financial risk, human errors, a non-fulfillment of contracts, an industrial risk, an environmental risk, etc. These problems were solved by means of the following methods: the scenario method [17], the multi-agent systems [1, 7, 19], the multi-criteria models [6, 27, 31]. Additionally, this problem was analyzed on various levels: the world market risk [11], the regional economic system risk [20, 24, 26], the firm's risk [2, 28], the technology operation risk [3, 8].

The risk control problems are related to various aspects, in particular, an assessment of the risk factors, a choice of the risk management method, a prediction of the damage, etc. Rasmussen and Svedung emphasized that «risk management can no longer be based on responses to past accidents and incidents, but must be increasingly proactive» [21]. Therefore, an importance of developing measures to prevent risks prevails over minimizing the damage from accidents.

Wu, Olson, and Choi indicated that «optimization and risk minimization inherently run counter to each other» [30]. Consequently, a choice of the risk management method should be based on an assessment of the preventive measures economic efficiency. These problems were solved on the basis of the multicriteria decision making (MCDM) methods [6, 27, 31], and the biconvex models and algorithms [25].

[*] Corresponding author: first@ras.ru

For MCDM, Heller [27] proposed using a pair wise comparison of the risk competing objectives. The following criteria were analyzed: a power outage, a fire, a flood, an earthquake, a hurricane, a destruction of buildings, network failures, etc. As a result, a matrix of the risk criteria assessments was formed, which is used in a qualitative risk analysis for three buildings that differ in qualitative features.

Abla et al. [6] used MCDM to derive an aggregated risk score on the basis of the fuzzy logic. For assessing development scenarios of the risk situation, the decision-making process was considered under the following criteria: a technical and functional efficiency, an economic sustainability, a social sustainability, an institutional environmental sustainability, an overall sustainability. The risk management strategy was selected based on a weighted sum of these criteria. In the case of the flood risk, this decision making technique was applied to select the most sustainable strategy under uncertainty.

Yazdani et al. [31] proposed MCDM type for assessing risks of the cultivated areas' flooding; they ranked various agricultural projects, which can mitigate the flood risks.

Dudin M. N. et al. [5] investigated the risks of an industrial enterprise and calculated external factors influence weights and a likelihood of unforeseen events for political, macroeconomic, social, and technological factors. On the basis of these weights, the external risk average level of Russian industrial enterprises was calculated. Internal risks of the enterprise were assessed according to the following criteria: a fulfillment of the production plan, an economic security of current obligations, an economic security of supply contracts, a human factor, and a likelihood of success in innovations commercialization. The authors examined hedging and insurance methods for the risk management of an industrial enterprise, however, the insurers and other related organizations were not considered as separate agents, and their interactions were not investigated.

Krokhina J.A. et al. [16] explored the environmental risks of industrial enterprises using a tree method and assessed the integral risk of an industrial facility as a result of its negative impact on an environment, a human health, and an enterprise's economy. The authors applied the tree method to assessing the risk of an accident on the main pipeline and assessed an economic efficiency of the measures to reduce the environmental risk of industrial enterprises, but did not take into account an interaction of the enterprise with other economic agents.

In contrast to the aforementioned literature, multi-agent systems (MAS) models expanded a range of the risk management techniques. MAS models were studied in production design and development systems, a production planning and management, and a supply chain management (SCM) [19]. In a set of agents, MAS models described the participants in the production process supply chain. In SCM, an influence of small and medium-sized enterprises in the interaction with large firms [7] and buyers with suppliers [1] on the risk level was considered. In the case of different-scale enterprises in SCM, Finch [7] emphasized varying degrees of the supply chain disruption risk, because large and small enterprises are subject to different degrees of the risks and their sensitivity to the risk factors is different. Ahn and Park [1] studied the information exchange processes between participants in the supply chain as MAS agents and assessed an impact of an agent's awareness on SCM. In general, MAS model was applied for the risk management in systems, which consist of an industrial enterprise and its suppliers, i.e., the participants in the supply chain.

Finally, it should be emphasized that the aforementioned studies were carried out for specific industries. For example, the problems of risk in the chemical and oil industries were solved [3, 8] and technical, human and organizational factors of the industrial facility risk were taken into account; on the basis of the fuzzy logic, the risk of the industrial facility in the oil and gas industry was analyzed using the criteria of frequency, detectability and damage value [30]. Therefore, the results of these studies cannot be applied to all industries.

Thus, we demonstrate the following research gap in the problem framework of the industrial risk management. On the one hand, the industrial researchers pointed to a need for the insurance and the technical measures to prevent or eliminate consequences of industrial accidents. On the other hand, they did not investigate the interaction mechanisms of an enterprise with insurers and waste utilization firms as the specific agents. Additionally, they did not generalize the results for the universal industrial enterprise; they were limited to the specific industry. At the same time, MAS researchers did not study the principles of applying MAS in the process of industrial risk management.

Hence, we can formulate the following research question of the industrial risk management problem: to describe the industrial risk management process for the universal industrial enterprise within the system of interconnected economic agents and calculate the equilibrium prices for services that circulate within this system.

Our study aims at calculating the price equilibrium in the system, which appears as a result of the preventive measures to minimize the consequences of technical accidents in the industry.

2. PROBLEM FRAMEWORK

In this paper, the risk is considered at the firm's level, and it includes an internal damage and an external damage. The internal damage causes a reduction in the firm's assets. The external damage is the property wastes of other firms, individuals and the environment. Additionally, the fiscal penalties (the ecology payment, the penalty for a damage to health, and a property of other firms and individuals, the compensation caused by the non-fulfillment contracts) depend on a value of the external damage. The industrial firm can reduce the internal/external damage by means of additional expenses on the risk reduction. These costs expenses are named the voluntary risk costs (VRC).

We analyze the problem of the industrial risk control for a system with three agents: the industrial firm, the waste utilization firm, and the insurance company. These agents are in various relationships in the system. We consider the following problem: to search for a compromise price of waste utilization and a compromise insurance rate, which are compliant with all agents of the system.

We assume that each participant in the system is intended to increase its profits, and he chooses the optimal price. If these optimal prices are different, then the participants may not agree to conclude a contract, then the industrial risk management will not be implemented. Therefore, in this case, we determine the set of possible values of the insurance rate and the price of waste utilization, at which the participants in the system will agree.

We introduce the following assumptions, which determine the applicability limits of the model.

Assumption 1.

The product price is an exogenous constant, that is, the firm does not affect the price

$$\frac{dp}{dQ} = 0, \tag{1}$$

where p is the price of the production, Q is the production volume.

The waste utilization firms and the insurance companies are in the monopolistic competition market, that is, the following conditions are fulfilled:

$$\frac{dp_Y}{dY^U} < 0, \quad \frac{dp_Y}{dX^U} < 0, \tag{2}$$

$$\frac{\partial T}{\partial X^S} < 0, \frac{\partial T}{\partial Y^S} < 0, \tag{3}$$

where p_Y is the price of the utilization of a conventional waste unit, T is the insurance rate, X^U and Y^U are the internal and external utilized damage, X^S and Y^S are the internal and external insured damage.

Assumption 2.

The production growth leads to a decreasing return:

$$C''_{QQ} < 0, \tag{4}$$

where C is a value of the firm's costs.

Assumption 3.

An increase in the production assets leads to an increasing in the possible damage; the internal damage and the external damage are reduced with an increase in VRC; the internal damage is limited from above due to technology features and the production volume

$$\frac{\partial X}{\partial Q} > 0, \frac{\partial X}{\partial f} < 0, X \in (0, X^{\max}], X^{\max} > 0. \tag{5}$$

where X^{max} is the maximum possible internal damage, X is the internal damage, f is VRC.

Assumption 4.

The external damage Y is proportional to the internal damage X:

$$\frac{\partial Y}{\partial X} > 0. \tag{6}$$

Assumption 5.

The voluntary combination insurance is considered, the wear is not included. The insurance indemnity W is proportional to the insured damage X^S and Y^S, the indemnity does not exceed the damage:

$$\frac{\partial W}{\partial X^S} > 0, \frac{\partial W}{\partial Y^S} > 0, \; W \le X^S + Y^S. \tag{7}$$

Assumption 6.

The cost of the utilization of a conventional waste unit c_Y is a constant.

$$c_Y = const. \tag{8}$$

Assumption 7.

The firm's external damage $Y=Y^S+Y^U+Y^{res}$ consists of the insured external damage $Y^S=\delta^S Y$, the utilized external damage $Y^U=\delta^U Y$, and the residual external damage $Y^{res}=\delta^{res} Y$. The firm's internal damage $X=X^S+X^U+X^{res}$ consists of the insured external damage $X^S=\gamma^S X$, the utilized internal damage $X^U=\gamma^U X$, and the residual internal damage $X^{res}=\gamma^{res} X$.

$$\delta^S + \delta^U + \delta^{res} = 1, \quad \delta^S, \delta^U, \delta^{res} \ge 0, \tag{9}$$

$$\gamma^S + \gamma^U + \gamma^{res} = 1, \quad \gamma^S, \gamma^U, \gamma^{res} \ge 0. \tag{10}$$

The production costs function, according to assumption 2, has the following form [4], [29].

$$C_Q(Q) = BQ^{\beta}, \; \beta \in (1, \beta^{\max}], \beta^{\max} \in (1, 2], B > 0, \tag{11}$$

where B and β are the parameters of the production costs function, $\beta^{\max}$ is the maximum possible parameter value.

The internal damage function satisfies assumption 3, and it has the following form:

$$X(Q, f) = \varphi(Q) e^{-\xi f}, \; \xi \in (0, \xi^{\max}], \xi^{\max} \in (0, 1], \; \varphi'(Q) > 0. \tag{12}$$

This function $X(Q)$ expresses an exponential distribution of the damage, which corresponds to man-made accidents, $\varphi(Q)$ is the dependence of the damage on the production volume Q, ξ is the parameters of the internal damage function, $\xi^{\max}$ is the maximum possible parameter value.

The external damage function satisfies assumption 4:

$$Y(X) = \mu X,\ \mu \geq 0\ . \tag{13}$$

The coefficient of the accident consequences expansion μ expresses the ratio of the external damage and the internal damage, taking into account the specifics of the industrial complex in a region, geographical features, etc.

The insurance indemnity satisfies assumption 5:

$$W(X^S, Y^S) = \alpha(X^S + Y^S),\ 0 \leq \alpha \leq 1, \tag{14}$$

where α is the coefficient of the insurance indemnity.

The penalty function has the following form:

$$H = aY = a\mu X,\ a > 0, \tag{15}$$

where a is the parameter of a relationship between the penalty and the external damage.

We consider the systems, which include the agents of three types: the 1st agent is the industrial firm, the 2nd agent is the waste utilization firm, and the 3rd agent is the insurance company.

The industrial firm (the 1st agent) produces the production volume Q, and it sells the product at the price p. We introduce the following notation: C_Q is the production costs function, X is the internal damage, Y is the external damage, f is VRC, $H(Y)$ is the penalty function, $F(X,Y)$ is the value of waste utilization costs, $V(X, Y)$ is the insurance premium, $W(X, Y)$ is the insurance indemnity, Q^{max} is the maximum possible production volume, f^{max} is the maximum possible VRC.

The revenue function of the 1st agent is

$$R = Qp + W\ . \tag{16}$$

The total costs function of the 1st agent is

$$C_\Sigma = C_Q + f + X\gamma^{res} + V + H + F\ . \tag{17}$$

The profit function of the 1st agent is

$$\Pi_I = R - C_\Sigma. \tag{18}$$

We formulate the problem of the firm's choice as follows: to search for the production volume and VRC function, which maximize the profit of the 1st agent, that is:

$$\{f^*, Q^*\} = \underset{f \in A_f, Q \in A_Q}{\arg\max}\ \Pi_I\ . \tag{19}$$

$$A_Q = \{Q \in R^+ : Q \leq Q^{\max}, Q^{\max} > 0\} \tag{20}$$

$$A_f = \{f(\bullet) \in R^+ : f(\bullet) \leq f^{\max}, f^{\max} \in (0, R_i)\} \tag{21}$$

$$\begin{cases} X = \varphi(Q)e^{-\xi f}, \\ Y = \mu X, \\ C_Q = BQ^\beta, \\ F = p_Y(Y\delta^U + X\gamma^U), \\ H = aY\delta^{res}, \\ W = \alpha(X\gamma^S + Y\delta^S), \\ V = T(X\gamma^S + Y\delta^S). \end{cases} \tag{22}$$

The waste utilization firm (the 2nd agent) has the following parameters. The utilized damage $Y\delta^U + X\gamma^U$ does not exceed the level $\overline{Y}$, the price p_Y does not exceed the level $\overline{p}_Y$, where $\overline{p}_Y$ is the maximum possible price, $\overline{Y}$ is the maximum possible waste utilization. Consequently, the inverse demand function of the 2nd agent, according to assumption 1, has the form: $p_Y = \overline{p}_Y - \frac{\overline{p}_Y}{\overline{Y}}(Y\delta^U + X\gamma^U)$. If $p_Y > 0$, then $\forall X, Y:\ Y\delta^U + X\gamma^U \leq \overline{Y}$

The profit function of the 2nd agent is

$$\Pi_{II} = (p_Y - c_Y)(Y\delta^U + X\gamma^U). \tag{23}$$

We formulate the problem of the choice of p_Y : to search for the price of the utilization of a conventional waste unit, which maximizes the profit of the 2nd agent, that is:

$$p_Y^* = \arg\max_{p_Y \in R^+} \Pi_{II} \tag{24}$$

$$p_Y = \overline{p}_Y - \frac{\overline{p}_Y}{\overline{Y}}(Y\delta^U + X\gamma^U). \tag{25}$$

The insurance company (the 3rd agent) has the following parameters. The insurance premium depends on the insurance rate T and the insured damage $Y\delta^S + X\gamma^S$; $\overline{T}$ is the maximum possible insurance rate, $\overline{X}$ is the maximum possible insured damage. Consequently, the inverse demand function of the 3rd agent, according to assumption 1, has the form: $T = \overline{T} - (Y\delta^S + X\gamma^S)\frac{\overline{T}}{\overline{X}}$. If $T > 0$, then $\forall X, Y:\ Y\delta^S + X\gamma^S \le \overline{X}$.

The profit function of the 3rd agent is

$$\Pi_{III} = V - W. \tag{26}$$

We formulate the problem of the choice of the insurance rate: to search for the insurance rate, which maximizes the profit of the 3rd agent, that is:

$$T^* = \arg\max_{Tst \in (0,1)} \Pi_{III} \tag{27}$$

$$\begin{cases} V = T(Y\delta^S + X\gamma^S), \\ T = \overline{T} - (Y\delta^S + X\gamma^S)\dfrac{\overline{T}}{\overline{X}}, \\ W = \alpha(Y\delta^S + X\gamma^S), \\ Y\delta^S + X\gamma^S \le \overline{X}. \end{cases} \tag{28}$$

We consider the following problem of the agent's optimal control: to search for the pair (Q^*, f^*), which is optimal according to criterion (19), to search for the price p_Y^*, which is optimal according to criterion (24), and to search for the rate T^*, which is optimal according to criterion (27).

This system consists of three agents, and, if we vary the parameters Q, f, p_Y, T, then three agents achieve maximums of their profits.

We consider the system of the industrial damage control of the following types. The agents have *a strong relation*, if they have the collective criterion function, and their costs are not separable. The agents have *a weak relation*, if the costs are separable, and each agent has the individual criterion function. We investigate the types of the system in the following models.

Model 1: the 1st agent is the customer of the waste utilization and the insurance, the 2nd agent is the contractor of the waste utilization, the 3rd agent is the insurer of the internal damage and the external damage of the 1st agent. This system has the weak relation type. (Fig. 1)

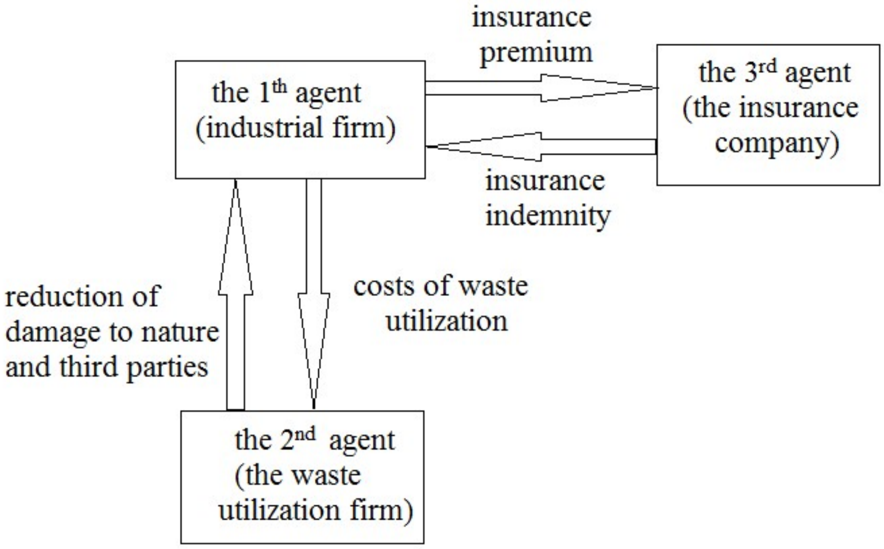

Fig. 1. The agent's interaction schema in Model 1.

The 1st agent pays the sum $F = p_Y (Y\delta^U + X\gamma^U)$ to the 2nd agent, and the agents achieve the contract, if the price p_Y complies with everyone. We formulate the problem with two criteria to search for the compromise price p_Y^{com}, under which the system is stable.

By using an analogy with the previous case, the 1st agent pays the sum $V = T(Y\delta^S + X\gamma^S)$ to the 3rd agent, and the agents achieve the contract, if the insurance rate T complies with everyone. We formulate the problem with two criteria to search for the compromise rate T^{com}, under which the system is stable.

Thus, the system of three agents is stable, if the compromise price p_Y^{com} and compromise rate T^{com} are indicated in the contracts.

Consequently, we formulate the following problems: to search for the compromise price p_Y^{com} and the compromise rate T^{com}, which satisfy the following conditions:

$$\max_{p_Y \in G} \Pi_I \wedge \max_{p_Y \in G} \Pi_{II}, \tag{29}$$

$$\max_{T \in \Omega} \Pi_I \wedge \max_{T \in \Omega} \Pi_{III}, \tag{30}$$

$$G = \{p_Y \mid \Pi_I(p_Y) > 0 \wedge \Pi_{II}(p_Y) > 0\}, \tag{31}$$

$$\Omega = \{T \mid T \in (0,1) \wedge \Pi_I(T) > 0 \wedge \Pi_{III}(T) > 0\}. \tag{32}$$

In formulas (29), (30), the symbol of the conjunction "$\wedge$" means that the maximums are determined according to both criteria, taking into account the Pareto optimal principle.

Model 2: the 1st agent and the 2nd agent have the strong relation; the 3rd agent is the insurer of the internal damage and the external damage of the 1st and the 2nd agents; the 3rd agent has the weak relation to other agents. (Fig. 2)

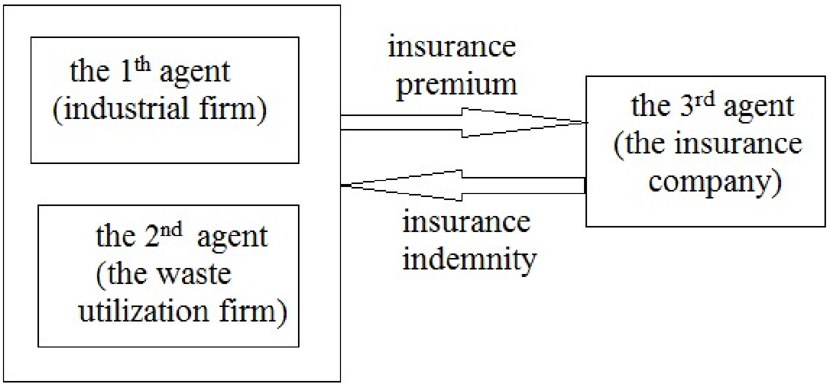

Fig. 2. The agent's interaction schema in Model 2.

The aggregate profit function of the 1st and the 2nd agents in the system is

$$\Pi_{I,II} = \Pi_I + \Pi_{II} = Qp + W - C_Q - f - V - H - c_Y(Y\delta^U + X\gamma^U) - X\gamma^{res} . \qquad (33)$$

The problem of searching for the optimal production volume and VRC for the 1st agent, and, additionally, the optimal rate of the 3rd agent, has the following form:

$$\{f^*, Q^*\} = \underset{f \in A_f, Q \in A_Q}{\arg\max} \ \Pi_{I,II} , \qquad (34)$$

$$\begin{cases} X = \varphi(Q) e^{-\xi f}, \\ Y = \mu X, \\ C_Q = BQ^{\beta}, \\ Y\delta^U + X\gamma^U \le \overline{Y}, \\ H = aY\delta^{res}, \\ W = \alpha(X\gamma^S + Y\delta^S), \\ V = T(X\gamma^S + Y\delta^S). \end{cases} \qquad (35)$$

These variables influence on the waste utilization, the insured damage, the penalties, and the insurance rate.

We formulate the problem of searching for the compromise rate accounting to the following conditions:

$$\max_{T \in \Omega_1} \Pi_{I,II} \wedge \max_{T \in \Omega_1} \Pi_{III} \ , \qquad (36)$$

$$\Omega_1 = \{T \mid T \in (0,1) \wedge \Pi_{I,II}(T) > 0 \wedge \Pi_{III}(T) > 0\} . \qquad (37)$$

Model 3: two agents of the 1st type are customers of the waste utilization and the insurance; the 2nd agent is the contractor of the waste utilization; the 3rd agent is the insurer of the internal damage and the external damage of the 1st agent. This system has the weak relation. (Fig 3)

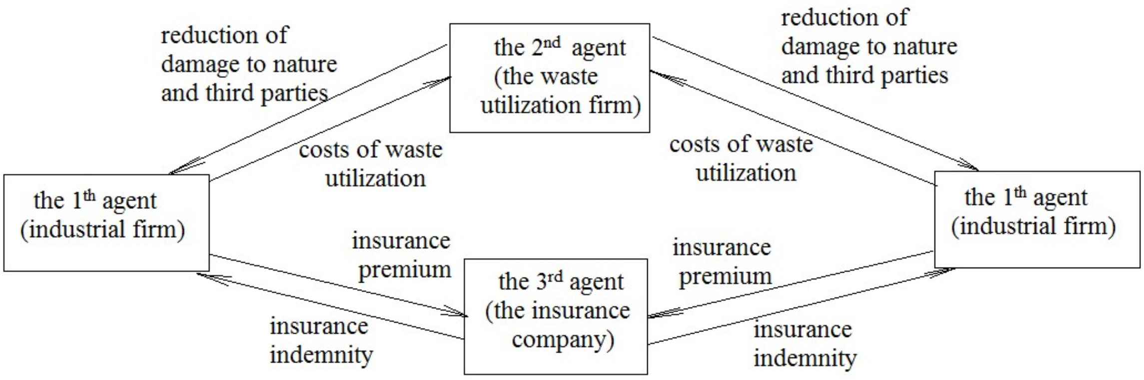

Fig. 3. The agent's interaction schema in Model 3.

The problem of searching for VRC and the optimal production volume of the 1st type agents is

$$\Pi_{I\,i} = Q_i p_i + W_i - C_{Q\,i} - f_i - X_i \gamma_i^{res} - V_i - H_i - F_i,\ \ i = \overline{1,2}, \tag{38}$$

$$\{f_i^*, Q_i^*\} = \underset{f_i \in A_f, Q_i \in A_Q}{\arg\max}\ \pi_I, \tag{39}$$

$$\begin{cases} X_i = \varphi_i(Q_i) e^{-\xi f_i}, \\ Y_i = \mu X_i, \\ C_{Q\,i} = B_i Q_i^{\beta_i}, \\ F_i = p_Y (Y_i \delta_i^U + X_i \gamma_i^U), \quad i = \overline{1,2} \\ H_i = a Y_i \delta_i^{res}, \\ W_i = \alpha_i (X_i \gamma_i^S + Y_i \delta_i^S), \\ V_i = T(X_i \gamma_i^S + Y_i \delta_i^S). \end{cases} \tag{40}$$

where $\pi_I = \{\Pi_{Ii}, i = 1,2\}$ is the vector of criteria in model 3.

The profit function (23) of the 2nd agent is the sum of the revenues, which are provided by two agents of the 1st type, and it has the following form:

$$\Pi_{II} = (p_Y - c_Y) \sum_{i=1}^{2} (Y_i \delta_i^U + X_i \gamma_i^U). \tag{41}$$

The problem of searching for the price p_Y* according to maximization of the 2nd agent's profit function is

$$p_Y^* = \underset{p_Y \in R^+}{\arg\max}\ \Pi_{II}, \tag{42}$$

$$p_Y = \overline{p}_Y - \frac{\overline{p}_Y}{\overline{Y}} (Y_i \delta_i^U + X_i \gamma_i^U),\ \ i = \overline{1,2}. \tag{43}$$

By using an analogy with the previous case, the profit function (26) of the 3rd agent is

$$\Pi_{III} = \sum_{i=1}^{2} (V_i - W_i). \tag{44}$$

The problem of searching for the insurance rate T according to the maximization of the 3rd agent's profit function is

$$T^* = \underset{Tst \in (0,\, 1)}{\arg\max}\ \Pi_{III} \tag{45}$$

$$\begin{cases} V_i = T(Y_i\delta_i{}^S + X_i\gamma_i{}^S), \\ T = \overline{T} - (Y_i\delta_i{}^S + X_i\gamma_i{}^S)\dfrac{\overline{T}}{\overline{X}}, \\ W_i = \alpha_i(Y_i\delta_i{}^S + X_i\gamma_i{}^S), \\ \sum\limits_{i=1}^{2}(Y_i\delta_i{}^S + X_i\gamma_i{}^S) \le \overline{X}. \end{cases} \tag{46}$$

We formulate the problem of searching for the compromise price p_Y^{com} and the compromise rate T^{com}, accounting to the following conditions:

$$\max_{p_Y \in G_1} \Pi_{I1} \wedge \max_{p_Y \in G_1} \Pi_{I2} \wedge \max_{p_Y \in G_1} \Pi_{II}\,. \tag{47}$$

$$G_1 = \{p_Y \mid \Pi_{I\,1}(p_Y) > 0 \wedge \Pi_{I\,2}(p_Y) > 0 \wedge \Pi_{II}\,(p_Y) > 0\}\,. \tag{48}$$

$$\max_{T \in \Omega_2} \Pi_{I1} \wedge \max_{T \in \Omega_2} \Pi_{I2} \wedge \max_{T \in \Omega_2} \Pi_{III} \tag{49}$$

$$\Omega_2 = \{T \mid T \in (0,1) \wedge \Pi_{I\,1}(T) > 0 \vee \Pi_{I\,2}(T) > 0 \wedge \Pi_{III}\,(T) > 0\}. \tag{50}$$

3. RESULTS

Assertion 1.

The function

$$f^* = \frac{1}{\xi}\ln|\,\xi\chi(Q)K\,|,$$

$$K = -\alpha\gamma^S - \mu\alpha\delta^S + \gamma^{res} + T\gamma^S + \mu T\delta^S + a\mu\delta^{res} + \mu\rho_Y\delta^U + \rho_Y\gamma^U$$

and the value Q, which is calculated from the equation* $p - B\beta Q^{\beta-1} - \dfrac{\varphi'(Q)}{\xi\varphi(Q)} = 0$, *are the solution of the problem (19 – 22) for the continuously differentiable functions* $\varphi(\cdot)$ *and under conditions*

$$\xi^2\varphi(Q)e^{-\xi f}K(B\beta(\beta-1)Q^{\beta-2} + \varphi''(Q)e^{-\xi f}K) - (\xi\varphi'(Q)e^{-\xi f}K)^2 > 0 \text{ and } K > 0,$$

$$\varphi''(Q)\varphi(Q) - \varphi'^2(Q) \ge 0\,.$$

The maximal profit of the 1st agent is

$$\Pi_I{}^* = Q^*p - BQ^{*\beta} - f^* - \frac{1}{\xi}\,.$$

Proof. The profit function of the 1st agent (18) is

$$\Pi_I = Qp + (\alpha - T)(X\gamma^S + Y\delta^S) - BQ^\beta - f - \chi(Q)e^{-\xi f}\gamma^{res} - aY\delta^{res} - \\ - p_Y(X\gamma^U + Y\delta^U)\,.$$

We search for the partial derivatives of this function, which are equal to zero:

$$\frac{\partial \Pi_I}{\partial f} = -1 - \xi\varphi(Q)e^{-\xi f}[(\alpha - T)(\gamma^S + \mu\delta^S) - \gamma^{res} - a\mu\delta^{res} - p_Y(\gamma^U + \mu\delta^U)] = 0$$

$$\frac{\partial \Pi_I}{\partial Q} = p - B\beta Q^{\beta-1} + \varphi'(Q)e^{-\xi f}[(\alpha - T)(\gamma^S + \mu\delta^S) - \gamma^{res} - a\mu\delta^{res} - p_Y(\gamma^U + \mu\delta^U)] = 0$$

We solve these equations as follows:

$$\xi\varphi(Q)e^{-\xi f}[(T - \alpha)(\gamma^S + \mu\delta^S) + \gamma^{res} + a\mu\delta^{res} + p_Y(\gamma^U + \mu\delta^U)] = 1$$

We introduce the denotation $K=(\gamma^S+\delta^S)(T-\alpha)+\gamma^{res}+a\mu\delta^{res}+p_Y(\mu\delta^U+\gamma^U)$, then we can write the optimal function VRI as follows: $f^*=\frac{1}{\xi}\ln(\xi\varphi(Q)K)$.

We write the equation

$p-B\beta Q^{\beta-1}-\varphi'(Q)e^{-\xi f}K=0$,

and, for the aforementioned symbols K and f^*, we get $p-B\beta Q^{\beta-1}-\frac{\varphi'(Q)}{\xi\varphi(Q)}=0$.

We solve this equation, and we search for Q^*.

We consider a function $h(Q)=p-B\beta Q^{\beta-1}-\frac{\varphi'(Q)}{\xi\varphi(Q)}$. This function is continuous function $\forall Q\in A_Q$. Due to the features of continuous functions, the equation $h(Q)=0$ has a solution, because

$$h(Q)<0 \text{ for } \forall Q\in A_Q \mid p<B\beta Q^{\beta-1}+\frac{\varphi'(Q)}{\xi\varphi(Q)},$$

$$h(Q)>0 \text{ for } \forall Q\in A_Q \mid p>B\beta Q^{\beta-1}+\frac{\varphi'(Q)}{\xi\varphi(Q)},$$

and

$$h'(Q)=-B\beta(\beta-1)Q^{\beta-2}-\frac{\varphi''(Q)\varphi(Q)-\varphi'^2(Q)}{\xi\varphi^2(Q)}<0 \text{ if } \varphi''(Q)\varphi(Q)-\varphi'^2(Q)\geq 0.$$

We check a fulfillment of the maximum sufficient condition for the function $\Pi_I(Q,f)$ at $f=f^*$ and $Q=Q^*$. For this purpose, we define the sign of $\Delta=\frac{\partial^2\Pi_I}{\partial f^2}\frac{\partial^2\Pi_I}{\partial Q^2}-\left(\frac{\partial^2\Pi_I}{\partial f\partial Q}\right)^2$.

$$\frac{\partial^2\Pi_I}{\partial f^2}=-\xi^2\varphi(Q)e^{-\xi f}K,$$

$$\frac{\partial^2\Pi_I}{\partial Q^2}=-B\beta(\beta-1)Q^{\beta-2}-\varphi''(Q)e^{-\xi f}K,$$

$$\frac{\partial^2\Pi_I}{\partial f\partial Q}=\xi\varphi'(Q)e^{-\xi f}K.$$

$$\Delta=\xi^2\varphi(Q)e^{-\xi f}K(B\beta(\beta-1)Q^{\beta-2}+\varphi''(Q)e^{-\xi f}K)-(\xi\varphi'(Q)e^{-\xi f}K)^2>0.$$

If $K>0$, then $\frac{\partial^2\Pi_I}{\partial f^2}<0$ and $\frac{\partial^2\Pi_I}{\partial Q^2}<0$, consequently, the pair (f^*,Q^*) is the maximum point according to Sylvester's criterion. ■

Assertion 2.

The value $p_Y^*=\frac{c_Y+\bar{p}_Y}{2}$ *is the solution to the problem (24 – 25) for the continuously differentiable function* $Y(p_Y)$.

The maximal profit of the 2nd agent is

$$\Pi_{II}^*=\Pi_{II}(p_Y^*)=\frac{1}{4\bar{p}_Y}(\bar{Y}(\bar{p}_Y-c_Y)^2).$$

Proof: We write the profit function of the 2nd agent (23), which is subjected to the condition (25):

$$\Pi_{II} = (p_Y - c_Y)(\overline{p}_Y - p_Y)\frac{\overline{Y}}{\overline{p}_Y}.$$

We transform this expression as follows:

$$\Pi_{II} = p_Y^2\left(-\frac{\overline{Y}}{\overline{p}_Y}\right) + p_Y\left(\overline{Y} + \frac{\overline{Y}}{\overline{p}_Y}c_Y\right) - \overline{Y}c_Y.$$

This expression is the second-order power function, and it has the parabola graph with the maximum at $p_Y* = \frac{\overline{p}_Y + c_Y}{2}$..

We search for the maximum of the profit function $\Pi_{II}* = \Pi_{II}(p_Y*)$ of the 2nd agent:

$$\Pi_{II}* = \left(\frac{\overline{p}_Y + c_Y}{2}\right)^2\left(-\frac{\overline{Y}}{\overline{p}_Y}\right) + \left(\frac{\overline{p}_Y + c_Y}{2}\right)\left(\overline{Y} + \frac{\overline{Y}}{\overline{p}_Y}c_Y\right) - \overline{Y}c_Y = \frac{\overline{Y}}{4\overline{p}_Y}(\overline{p}_Y - c_Y)^2. \blacksquare$$

Assertion 3.

The insurance rate $T* = \frac{\overline{T} + \alpha}{2}$ *is the solution to the problem (27 – 28) for the continuously differentiable functions X(T) and Y(T).*

The maximal profit of the 3rd agent is

$$\Pi_{III}* = \Pi_{III}(T*) = \frac{1}{4\overline{T}}\overline{X}(\overline{T} - \alpha)^2.$$

Proof: The profit function of the 3rd agent is

$$\Pi_{III} = V - W = (T - \alpha)(Y\delta^S + X\gamma^S) = (T - \alpha)\overline{X}\frac{\overline{T} - T}{\overline{T}}.$$

We transform this function as follows:

$$\Pi_{III} = T^2\left(-\frac{\overline{X}}{\overline{T}}\right) + T\left(\overline{X} + \frac{\alpha\overline{X}}{\overline{T}}\right) - \alpha\overline{X}.$$

This function has the parabola graph with the maximum point at $T* = \frac{\overline{T} + \alpha}{2}$.

We determine the maximum of the profit function of the 3rd agent as follows:

$$\Pi_{III}* = \Pi_{III}(T*) = \left(\frac{\overline{T} + \alpha}{2}\right)^2\left(-\frac{\overline{X}}{\overline{T}}\right) + \left(\frac{\overline{T} + \alpha}{2}\right)\left(\overline{X} + \alpha\frac{\overline{X}}{\overline{T}}\right) - \alpha\overline{X} = \frac{\overline{X}}{4\overline{T}}(\overline{T} - \alpha)^2. \blacksquare$$

Assertion 4.

If $c_Y < \frac{\overline{p}_Y}{2}$, *then* $p_Y^{com} \in \left[c_Y; \frac{\overline{p}_Y}{2}\right]$ *is the solution of the problem (29), (31) for the continuously differentiable functions* $\varphi(\cdot)$, *else* $p_Y^{com} \in \varnothing$.

Proof: The profit function of the 1st agent is

$$\Pi_I(p_Y) = Qp + W - C_Q - f - X\gamma^{res} - V - H - p_Y(X\gamma^U + Y\delta^U)$$

The utilized damage $X\gamma^U + Y\delta^U$ corresponds to the demand function:

$$X\gamma^U + Y\delta^U = \overline{Y} - \overline{Y}\frac{p_Y}{\overline{p}_Y}.$$

Then the profit function of the 1st agent is

$$\Pi_I(p_Y) = K_1 - p_Y\overline{Y}\left(1 - \frac{p_Y}{\overline{p}_Y}\right) = p_Y^2\frac{\overline{Y}}{\overline{p}_Y} - p_Y\overline{Y} + K_1,$$

where K_1 is $Qp + W - C_Q - f - X\gamma^{res} - V - H$.

The maximum point of this function is $p_Y = \frac{\overline{p}_Y}{2}$, and the maximum of the profit function of the 1st agent $\Pi_I(p_Y)$ is $\Pi_I\left(\frac{\overline{p}_Y}{2}\right) = -\frac{\overline{p}_Y \overline{Y}}{4} + K_1$.

The profit function $\Pi_{II}(p_Y)$ is analyzed in the proof of assertion 2.

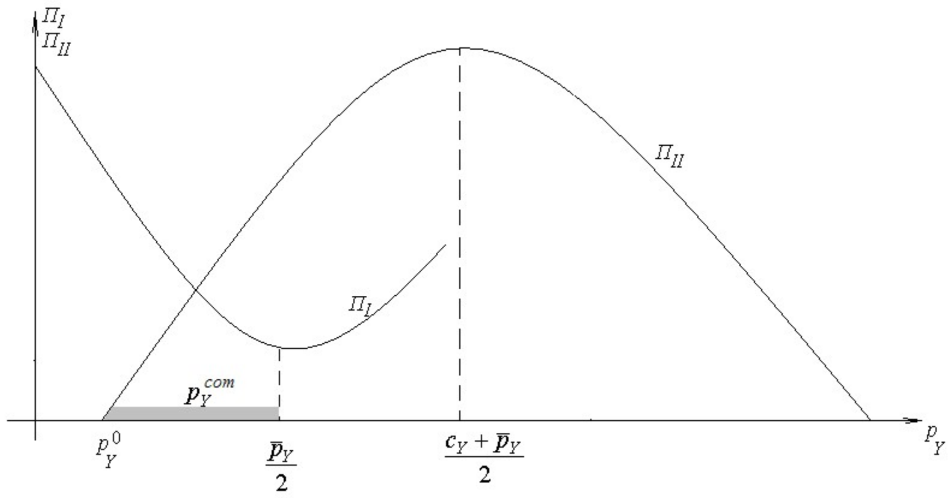

Fig. 4. Compromise set of the 1st and the 2nd agents

The value p_Y^0 is c_Y.

The figure 4 shows that the 1st agent prefers price $p_Y \in \left[0, \frac{\overline{p}_Y}{2}\right]$, the 2nd agent prefers price $p_Y \in \left[p_Y^0, \frac{c_Y + \overline{p}_Y}{2}\right]$. If $p_Y^0 \le \frac{\overline{p}_Y}{2}$ then $p_Y^{com} \in \left[p_Y^0, \frac{\overline{p}_Y}{2}\right]$ else $\left[p_Y^0, \frac{c_Y + \overline{p}_Y}{2}\right] \cap \left[0, \frac{\overline{p}_Y}{2}\right] = \varnothing$ ■

The value p_Y^{com} is the solution of the problem (29), (31), and it enables us to establish the price p_Y, which complies with the 1st and the 2nd agents. If one of the agents changes this price, then the profit of other agent decreases, therefore, this is Pareto optimal equilibrium set for 1st and the 2nd agents' prices according to profit function (18) as the criterion of the 1st agent and profit function (23) as the criterion of the 2nd agent.

Assertion 5.

If $\alpha < \frac{\overline{T}}{2}$, *then* $T^{com} \in \left[\alpha; \frac{\overline{T}}{2}\right]$ *is the solution of the problem (30), (32) for the continuously differentiable functions* $\varphi(\cdot)$, *else* $T^{com} \in \varnothing$.

Proof: The profit function 1st agent is

$$\Pi_I(T) = Qp + W - C_Q - f - X\gamma^{res} - H - F - T(X\gamma^S + Y\delta^S).$$

The insured damage $X\gamma^S + Y\delta^S$ corresponds to the demand function in the insurance market: $X\gamma^S + Y\delta^S = \overline{X} - \overline{X}\dfrac{T}{\overline{T}}$.

We transform the profit function of 1st agent as follows:

$$\Pi_I(T) = K_2 - T\overline{X}\left(1 - \frac{T}{\overline{T}}\right) = T^2\frac{\overline{X}}{\overline{T}} - T\overline{X} + K_2.$$

The value K_2 is $Qp + W - C_Q - f - X\gamma^{res} - F - H$.

The minimum point of profit function $\Pi_I(T)$ is $T = \dfrac{\overline{T}}{2}$ and the minimum of the function is $\Pi_I\left(\dfrac{\overline{T}}{2}\right) = -\dfrac{\overline{T}\overline{X}}{4} + K_2$.

The profit function $\Pi_{III}(T)$ is analyzed in the proof of assertion 3.

The graphs of the 1st and the 3rd agents' profits for problem (30), (32) are demonstrated in Fig. 5.

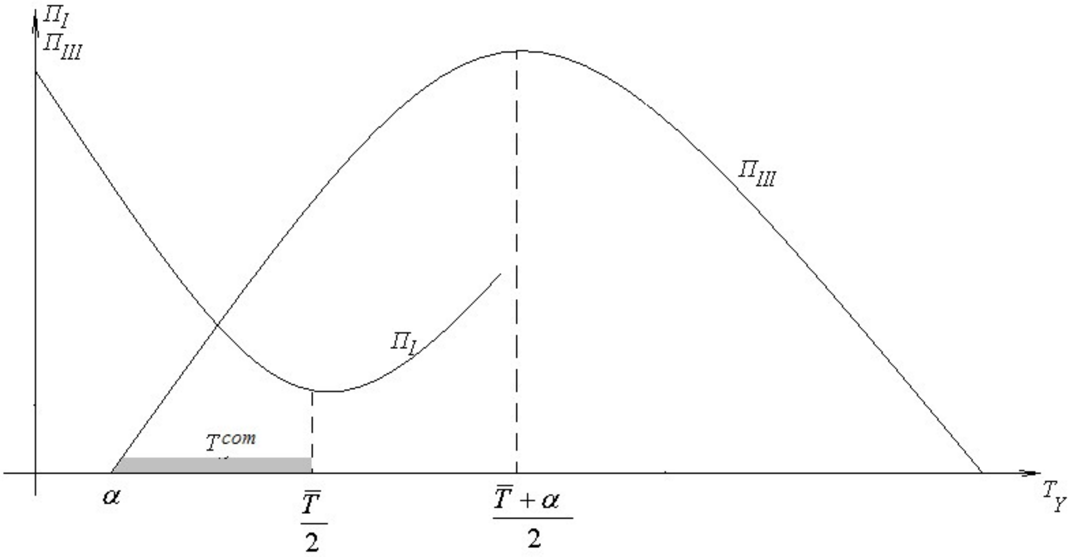

Fig. 5. Compromise set of the 1st and the 3rd agents

The acceptable set T^{com} is interval $\left[\alpha;\ \dfrac{\overline{T}}{2}\right]$. These values enable us to transact of the insurance contract, because the value T^{com} complies with the 1st and the 3rd agents. The deviation of the insurance rate relative to T^{com} leads to a decrease in the profit of one of the agents.

Thus, we identify the set of the possible values of the insurance rate T^{com} and the price of waste utilization p_Y^{com}, at which the system agents interact, and the process of the industrial risk management is implemented.

Similar to the conclusion from assertion 4, we establish the Pareto optimal equilibrium set for the 1st and the 3rd agents' prices according to profit function (18) as the criterion of the 1st agent and profit function (26) as the criterion of the 3rd agent.

4. DISCUSSION

The currently accepted risk management model is based on the standards [13] - [15] and, in fact, implements the intra-firm management [3], [4], [6], [8], [16], [27], [31]. In comparison with the aforementioned literature, we investigate the process of the industrial risk management in the system of several organizations interconnected within the framework of this process. Our system consists of three agents (the industrial firm, the insurer, and the waste utilization firm) and provides a comprehensive examination of the risk management process.

In contrast to use of MAS for SCM, we consider the MAS model, which includes the agents that are not linked in a supply chain. We add the insurers and the waste utilization firm to this model; therefore, our results extend the studies [18], [23]. In addition, our results are industry invariant; consequently, it can be used for any industrial production. This generality of results distinguishes our study from authors who considered risks for a particular industry, for example [3], [8].

We derive an analytical form of the preventive risk costs function for an industrial enterprise and prove that these costs depend logarithmically on the production volume of an enterprise. This pattern means that the preventive risk costs increase more slowly in comparison with a growth of the production. This feature encourages enterprises to take the preventive measures and develops the concept of the proactive risk management [21]. Therefore, we prove that the proactive risk management strategy is optimal, i.e., it corresponds to the minimum costs.

We calculate the optimal values of the waste utilization price and the insurance rate, which maximize the objective functions of the waste utilization firm and the insurer. In comparison with the approach [4], [16], which is based on the exogenously specified insurance rate, in our model, the insurance rate is determined endogenously. In other words, our model is based on such values of the waste utilization price and the insurance rate that induce these firms to participate in contracts with the industrial enterprise. Consequently, in these conditions, the decentralized decision-making system is configured according to the mechanism of the centralized system.

In addition, we find the conditions of the compromise domain, i.e., the ranges of the waste utilization price and the insurance rate, within which the system participants are interested in concluding contracts. Therefore, we expand the MAS approach [1, 7, 19] related to the revenue sharing, and reformulated it into the mechanism of the price sharing contract. Next, we prove that the contract prices within the specified ranges are Pareto efficient. Therefore, when the price varies within the compromise domain, the profit of one participant grows, and the profit of the counterparty decreases, i.e., the price sharing contract corresponds to the profit sharing contract. This is an important advantage of our approach: our model not only allows us to estimate the damage from an industrial accident, but also to calculate the economic effects of all participants in the risk management process and choose the preventive costs sum that corresponds to the optimal solutions for all participants.

The results of this article can be used by industrial enterprises, insurance companies, and waste utilization firms to determine the insurance rate and the utilization price. The resulting compromise values of the rate and the price demonstrate the set of acceptable values at which a contract will be concluded.

Finally, we briefly outline the directions for further research within our version of the MAS model. Our results are obtained under certain restrictions on the type of a market (assumption 1), the production function (assumption 2), and the damage function (assumption 3). In the future, we plan to expand the study to consider other types of markets and other production and damage functions.

5. CONCLUSION

The optimization problems of searching for the firms' risk prevention costs were investigated in our previous articles [9], [22]. In particular, the optimal VRC function in the case of the internal damage prevention according to the condition of the firm's profit maximization was derived. The problem of the external damage control was analyzed, and the optimal VRC function with regard to fiscal penalties for the environmental damage and the civil penalties for individuals' property damages was proved. The problem of searching for the optimal risk costs, taking into account the reinvestment of the firm's profit, was considered, and the optimal VRC function for a firm's activity in the consequent periods was obtained. In this paper, the problem of the industrial risk control in different system's structures is investigated.

We consider a risk control system of an industrial firm, which includes the insurance company and the waste utilization firm. This system enables us to determine the conditions of the insurance contract and the waste utilization contract. The problem of the industrial risk control is a problem of the agents' interests congruence. The calculated values of the insurance rate and the waste utilization price are determined as the Pareto equilibrium set for price and insurance rate.

We obtain the following results. The problem of the firm's choice of the product volume and VRC function in the system, which includes an industrial firm, a waste utilization firm, and an insurance company, is solved in assertion 1. The problem of searching for the price of the utilization, which maximizes the profit of a waste utilization firm, is solved in assertion 2. The problem of searching for the insurance rate, which maximizes the profit of an insurance company, is solved in assertion 3. The problem of searching for the compromise utilization price and the compromise insurance rate, taking into account Pareto optimal principle for this variable, is solved in assertion 5. The final result provides the set of acceptable values at which a contract will be concluded.

REFERENCES

1. Ahn, H.J., Park, S.J. (2003) Modeling of a Multi-agent System for Coordination of Supply Chains with Complexity and Uncertainty. In: Lee J., Barley M. (eds) Intelligent Agents and Multi-Agent Systems. PRIMA 2003. Lecture Notes in Computer Science, Vol. 2891. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-39896-7_2
2. Arena, M., Arnaboldi, M. & Azzone, G. (2011) Is Enterprise Risk Management Real?, J*ournal of Risk Research*, 14, 779 – 797.
3. Bouloiz, H., Garbolino, E. (2019) System Dynamics Applied to the Human, Technical and Organizational Factors of Industrial Safety. Safety Dynamics (p. 93 – 106). Cham: Springer.
4. Choi, T. M., Chan, H. K., Yue, X. (2016) Recent Development in Big Data Analytics for Business Operations and Risk Management // *IEEE transactions on cybernetics*, Vol. 47, No. 1, 81-92.
5. Dudin, M.N., Frolova, E.E., Lubenets, N.A., Sekerin, V.D., Bank, S.V., Gorohova, A.E. (2016) Methodology of Analysis and Assessment of Risks of the Operation and Development of Industrial Enterprises // *Calitatea*, Vol. 17, No. 153, 53.
6. Edjossan-Sossou, A.M., Galvez, D., Deck, O., Heib, M.A., Verdel, T., Dupont, L., Chery, O., Camargo, M., Morel, L. (2020) Sustainable Risk Management Strategy Selection Using a Fuzzy Multi-Criteria Decision Approach // *International Journal of Disaster Risk Reduction*. Vol. 45, May 2020, 101474 https://doi.org/10.1016/j.ijdrr.2020.101474

7. Finch, P. (2004) Supply Chain Risk Management // *Supply Chain Management*, Vol. 9 No. 2, 183-196. https://doi.org/10.1108/13598540410527079
8. Gallab, M., Bouloiz, H., Youssef, L.A., Tkiouat, M. (2019) Risk Assessment of Maintenance Activities Using Fuzzy Logic // *Procedia Computer Science*, Vol. 148, 226-235
9. Geraskin, M., Rostova, E. (2018) Costs Function Optimization for Prevention Costs Function Optimization for Prevention of Firm's Industrial Risks with Regard to Reinvestment of Profit, *Advances in Systems Science and Applications*, Vol. 18, No. 4, 52-63.
10. Gorecki, S. et al.(2019) Risk Management and Distributed Simulation in Papyrus Tool for Decision Making in Industrial Context //*Computers & Industrial Engineering*, Vol. 137, 106039.
11. Hanson, D. & White, R (2004) Regimes of Risk Management in Corporate Annual Reports: a Case Study of One Globalizing Australian Company, *Journal of Risk Research,* 7, 445 – 460.
12. Hay, D., Morris, D. (1991) *The Theory of Industrial Organization,* Oxford University Press: Revised edition.
13. ISO 31000:2009 «Risk management – Principles and guidelines », 2009
14. ISO Guide 73:2009 «Risk management – Vocabulary», 2009
15. ISO/IEC 31010:2009 «Risk management – Risk assessment techniques»
16. Krokhina, J. A. et al. (2018) Environmental Risk Management System Projecting of Industrial Enterprises, Ekoloji, Vol. 27, No. 106, 735-744.
17. Kulba, V., Schelkov, A., Chernov, I., Zaikin, O. (2016) Scenario Analysis in the Management of Regional Security and Social Stability, *Intelligent Systems Reference Library*, 98, 249 – 268.
18. Lee, J., Lee, D. K. (2018) Application of Industrial Risk Management Practices to Control Natural Hazards, Facilitating Risk Communication, *ISPRS International Journal of Geo-Information*, Vol 7, № 9, 377
19. Lee, J.& Kim, C. (2008) Multi-Agent Systems Applications in Manufacturing Systems and Supply Chain Management: a Review Paper, *International Journal of Production Research,* Vol. 46, Issue 1, 233-265. https://doi.org/10.1080/00207540701441921
20. Pazdnikova, N.P., Shipitsyna, S. Y. (2014) Stress Analysis in Managing the Region's Budget Risks, Risk Factors for the Regional Economic Growth, *Economy of Region*, 3(39), 208 – 217.
21. Rasmussen J., Svedung, I. Proactive Risk Management in a Dynamic Society. – Swedish Rescue Services Agency, 2000.
22. Rostova, E.P., Geraskin M.I. (2018) Optimization of Costs Function for Prevention of Firms' Industrial Risks With Penalties. *The Proceedings of the Third Workshop on Computer Modeling in Decision Making (CMDM 2018). ACSR-Advances in Computer Science Research.* Vol. 85, 26-30.
23. Samanlioglu, F. (2013) A Multi-Objective Mathematical Model for the Industrial Hazardous Waste Location-Routing Problem, *European Journal of Operational Research*, Vol. 226, No. 2, 332-340
24. Sapiro, E.S., Miroljubova, T.V. (2008) Risk Factors for the Regional Economic Growth, *Economy of Region*, 1, 39 – 49.
25. Sherali, H.D., Alameddine, A., Glickman, T.S. (1994) Biconvex Models and Algorithms for Risk Management Problems, *American Journal of Mathematical and Management Sciences*, Vol. 14, No. 3-4, 197-228.
26. Shorikov, A.F. (2012) Dynamic Model of Minimax Control Over Economic Security State of the Region in the Presence of Risks, *Economy of Region*, 2(30), 258 – 266.

27. Heller, S. (2006) Managing Industrial Risk—Having a Tested and Proven System to Prevent and Assess Risk // *Journal of Hazardous Materials*. Vol. 130, Issues 1–2, 17 March, 58-63 https://doi.org/10.1016/j.jhazmat.2005.07.067
28. Thun, J.-H., Drüke, M. & Hoenig, D. (2011) Managing Uncertainty – an Empirical Analysis of Supply Chain Risk Management in Small and Medium-Sized Enterprises, *International Journal of Production Research*, 49, 5511 – 5525.
29. Walters, A.A. (1963). Production and Cost Functions: an Econometric Survey. Econometrica. The Econometric Society, Econometrica.
30. Wu, D.D., Olson, D.L., Choi T.M. (2017) Guest Editorial Special Issue on Risk Analytics in Industrial Systems, *IEEE Systems Journal*. Vol 11, №. 3, 1476-1478.
31. Yazdani, M., Gonzalez, E.D.R.S. and Chatterjee, P. (2019), A Multi-Criteria Decision-Making Framework for Agriculture Supply Chain Risk Management Under a Circular Economy Context, *Management Decision*, Vol. ahead-of-print No. ahead-of-print. https://doi.org/10.1108/MD-10-2018-1088