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Enhanced Results on Stability Criteria for Linear Time Delay Systems with Distributed Delay via Relaxed Double Integral Inequality

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Abstract: This paper investigates the matter of stability criteria for linear time delay systems with distributed delay. Firstly, a relaxed double integral inequality is established to estimate the double integral terms appearing within the derivative of Lyapunov-Krasovskii functionals (LKFs) with a triple integral term. Unlike the recently introduced Jensen's inequalities, Wirtinger based integral inequalities, refined Jensen's inequalities and therefore the auxiliary function based integral inequalities the proposed relaxed integral inequality provides large feasible solution region and fewer conservative results. Secondly, by constructing an augmented Lyapunov-Krasovskii functional with a triple integral term, the robust stability criteria for linear time delay systems with distributed delay are given in terms of linear matrix inequalities (LMIs), which may be easily computed by the LMI toolbox of MATLAB. Finally, two numerical examples are performed to indicate the effectiveness of the proposed criterion.

Keywords: robust stability, delay-dependent stability, Lyapunov functional, linear matrix inequalities, time-delay systems, distributed delay

1. INTRODUCTION

Many dynamic systems in the real world inevitably have time delays, and such delays often cause poor performance, oscillation or even instability of the system. Consequently, the stability issue of time-delay systems has attracted researchers for many years. The main attention is paid to determine the admissible delay region, for which the systems remain stable, by developing effective delay-dependent stability criteria via the Lyapunov-Krasovskii stability theory. The issue relies on the handling of the integral terms arising within the derivative of the Lyapunov-Krasovskii functionals (LKFs). The development of new methods for this problem has always been a very important consideration.

So as to scale back conservatism of stability criteria, variety of techniques are presented, including for example, Jensen's integral inequality [1], the Wirtinger-based integral inequality [2], the various forms of Wirtinger-based double integral inequality [3-5], Bessel-Legendre inequality [6], auxiliary function-based integral inequalities [7,8], the free-weighting matrix approach [4,9-11], reciprocally convex method [12,13] and free matrix based multiple integral inequality [14]. The well-known Jensen's inequality is often adopted because it could lead on to a stability test with fewer matrix variables. Recently, Wirtinger integral inequality introduced in [2] is shown more powerful than Jensen's inequality. Later, another forms of integral inequalities are reported in [6,7,15,16-19] to further reduce the conservatism.

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This paper presents a new relaxed double integral inequality to estimate the double integral term within the derivative of Lyapunov-Krasovskii functionals(LKFs). A replacement stability criterion is established by applying the newly proposed inequality which provides less conservatism and fewer number of decision variables. To indicate the effectiveness of the proposed criterion, two numerical examples are provided.

Notations: Throughout this paper, R^n is the n-dimensional Euclidean space and $R^{n\times n}$ is the set of all $n \times n$ real matrices. $X > 0 (X \geq 0)$ means that the matrix X is a real symmetric positive definite matrix (positive semi definite). I denote the identity matrix with appropriate dimensions; $col\{\cdot\}$ means a column vector. $*$ in a matrix represents the elements below the main diagonal of a symmetric matrix. The superscript T denotes the transpose of the matrix.

2. PROBLEM FORMULATION

Consider the following system with state and distributed delays:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1 x(t-h) + A_2 \int_{t-h}^{t} x(s)ds, \qquad (2.1) \\ x(t) &= \phi(t), t \in [-h, 0] \end{aligned}$$

where, $x(t) \in \mathrm{R}^n$ is the state vector, $A, A_1, A_2 \in \mathrm{R}^{n\times n}$ are constant matrices, h is a constant time delay satisfying $h > 0$ and $\phi(t)$ is a continuous vector-valued initial condition.

2.1. Lemma[20]:

For a given matrix $M > 0$, the following inequality holds for all continuously differentiable functions $x : [a, b] \to R^n$:

$$(b-a)\int_a^b \dot{x}(s)^T M \dot{x}(s)ds \geq \Omega_1^T M \Omega_1 + 3\Omega_2^T M \Omega_2 + 5\Omega_3^T M \Omega_3 + 7\Omega_4^T M \Omega_4$$

where

$$\begin{aligned} \Omega_1 &= x(b) - x(a) \\ \Omega_2 &= x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds \\ \Omega_3 &= x(b) - x(a) + \frac{6}{b-a}\int_a^b x(s)ds - \frac{12}{(b-a)^2}\int_a^b\int_\theta^b x(s)dsd\theta \\ \Omega_4 &= x(b) + x(a) - \frac{12}{b-a}\int_a^b x(s)ds + \frac{60}{(b-a)^2}\int_a^b\int_\theta^b x(s)dsd\theta \\ &- \frac{120}{(b-a)^3}\int_a^b\int_\sigma^b\int_\theta^b x(s)dsd\theta d\sigma \end{aligned}$$

Relaxed Double Integral Inequality

2.2. Lemma:

For symmetric positive definite matrix $z \in R^{n\times n}$, scalars $\alpha < \beta$, and vector $\phi : [\alpha, \beta] \to R^n$ such that the integration concerned is well defined, the following inequality holds:

$$\int_\alpha^\beta\int_u^\beta \varphi^T(s)Z\varphi(s)dsdu \geq \frac{2}{(\beta-\alpha)^2}\Omega_5^T Z\Omega_5 + \frac{16}{(\beta-\alpha)^2}\Omega_6^T Z\Omega_6 \qquad (2.2)$$

where

$$\begin{aligned}\Omega_5 &= \int_\alpha^\beta \int_u^\beta \varphi(s)dsdu \\ \Omega_6 &= -\int_\alpha^\beta \int_u^\beta \varphi(s)dsdu + \frac{3}{\beta-\alpha}\int_\alpha^\beta \int_u^\beta \int_\theta^\beta \varphi(s)dsd\theta du\end{aligned}$$

Proof: For a function $\lambda(s) = k_1 + k_2 s$, integration by parts, we have

$$\int_\alpha^\beta \int_u^\beta \lambda(s)\varphi(s)\, dsdu = \lambda(a)\int_\alpha^\beta \int_u^\beta \varphi(s)\, dsdu + 2k_2 \int_\alpha^\beta \int_u^\beta \int_\theta^\beta \varphi(s)dsd\theta du$$

By setting $\lambda(a) = -1, 2k_2 = \frac{3}{\beta-\alpha}$, the above equality is rewritten as

$$\int_\alpha^\beta \int_u^\beta \lambda(s)\varphi(s)\, dsdu = \Omega_6$$

Then the following equality is obtained for any vector Ω_0 and any matrix $M > 0$:

$$\int_\alpha^\beta \int_u^\beta \lambda(s)\Omega_0^T M\varphi(s)\, dsdu = \Omega_0^T M\Omega_6$$

Similarly, the following equalities are derived:

$$\begin{aligned}\int_\alpha^\beta \int_u^\beta \Omega_0^T L\varphi(s)\, dsdu &= \Omega_0^T L\Omega_5 \\ \int_\alpha^\beta \int_u^\beta \Omega_0^T LR^{-1}L^T\Omega_0\, dsdu &= \frac{(\beta-\alpha)^2}{2}\Omega_0^T LR^{-1}L^T\Omega_0 \\ \int_\alpha^\beta \int_u^\beta \Omega_0^T LR^{-1}M^T\lambda(s)\Omega_0\, dsdu &= 0 \\ \int_\alpha^\beta \int_u^\beta \lambda^2(s)\Omega_0^T MR^{-1}M^T\lambda(s)\Omega_0\, dsdu &= \frac{(\beta-\alpha)^2}{16}\Omega_0^T MR^{-1}M^T\Omega_0\end{aligned}$$

Using the above equalities and the schur complement derives the following equality:

$$\begin{aligned}&\int_\alpha^\beta \int_u^\beta \begin{bmatrix}\Omega_0 \\ \lambda(s)\Omega_0 \\ \varphi(s)\end{bmatrix}^T \begin{bmatrix} LZ^{-1}L^T & LZ^{-1}M^T & L \\ * & MZ^{-1}M^T & M \\ * & * & Z\end{bmatrix} \begin{bmatrix}\Omega_0 \\ \lambda(s)\Omega_0 \\ \varphi(s)\end{bmatrix} dsdu \\ &\quad = \int_\alpha^\beta \int_u^\beta \varphi^T(s)R\varphi(s)dsdu + Sym\left\{\Omega_0^T L\Omega_5 + \Omega_0^T M\Omega_6\right\} \\ &\qquad + \frac{(\beta-\alpha)^2}{2}\Omega_0^T\left\{\frac{8LZ^{-1}L^T + MZ^{-1}M^T}{8}\right\}\Omega_0 \geq 0.\end{aligned}$$

where

$$\Omega_0^T = \begin{bmatrix}\Omega_5^T & \Omega_6^T\end{bmatrix}, \quad L = \frac{-2}{(\beta-\alpha)^2}[Z \quad 0]^T \text{ and } \quad M = \frac{-16}{(\beta-\alpha)^2}[0 \quad Z]$$

that is,

$$\Omega_0^T L = \frac{-2}{(\beta-\alpha)^2}\Omega_5^T Z \quad \text{and} \quad \Omega_0^T M = \frac{-16}{(\beta-\alpha)^2}\Omega_6^T Z$$

which leads to (2.2).
This completes the proof.

2.3. Remark:

The proposed relaxed double integral inequality provides the tightest estimation value of the double integral term $\int_a^b \int_\theta^b x^T(s)Zx(s)dsd\theta > 0$, compared with the widely used Jensen's integral inequality and Wirtinger-based integral inequality. Moreover, the additional positive term $\frac{16}{(\beta-\alpha)^2}\Omega_6^T Z\Omega_6$ reduces the estimation gap. Therefore, the proposed relaxed double integral inequality will cause less conservative than the prevailing ones within the literature. By setting $\varphi(s) = \dot{x}(s)$, the subsequent lemma are often obtained from the above lemma 2.2.

2.4. Lemma:

For symmetric positive-definite matrix $Z \in R^{n\times n}$, scalars $\alpha < \beta$, and vector $\dot{x} : [\alpha, \beta] \to R^n$ such that the integration concerned is well defined, the following inequality holds:

$$\int_\alpha^\beta \int_u^\beta \dot{x}^T(s)Z\dot{x}(s)dsdu \geq 2\chi_1^T Z\chi_1 + 16\chi_2^T Z\chi_2$$

where

$$\begin{aligned}
\chi_1 &= x(\beta) - \frac{1}{\beta-\alpha}\int_\alpha^\beta x(s)ds \\
\chi_2 &= \frac{-1}{2}x(\beta) - \frac{1}{\beta-\alpha}\int_\alpha^\beta x(s)ds + \frac{3}{(\beta-\alpha)^2}\int_\alpha^\beta \int_u^\beta x(s)dsdu
\end{aligned}$$

3. MAIN RESULTS

In this section, delay dependent stability criteria for the system with distributed delays are derived interms of LMI as follows:

3.1. Theorem:

Given $h > 0$, the system (2.1) is assymptotically stable if there exists positive definite matrices $P \in R^{4n\times 4n}, Q, S, Z \in R^{n\times n}$, such that the following LMI holds:

$$\Xi = \Gamma P \Upsilon^T + \Upsilon P \Gamma^T + \Psi < 0 \tag{3.3}$$

where

$$\begin{aligned}
\Gamma &= [e_1 \quad e_3 \quad e_4 \quad e_5] \\
\Upsilon &= \begin{bmatrix} e_0 & e_1 - e_2 & he_1 - e_3 & \frac{h^2}{2}e_1 - e_4 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\Psi &= e_1Qe_1^T - e_2Qe_2^T + h^2e_0Se_0^T + \frac{h^2}{2}e_0Ze_0^T \\
&- (e_1-e_2)S(e_1-e_2)^T - 3\left(e_1+e_2-\frac{2}{h}e_3\right)S\left(e_1+e_2-\frac{2}{h}e_3\right)^T \\
&- 5\left(e_1-e_2+\frac{6}{h}e_3-\frac{12}{h^2}e_4\right)S\left(e_1-e_2+\frac{6}{h}e_3-\frac{12}{h^2}e_4\right)^T \\
&- 7\left(e_1+e_2-\frac{12}{h}e_3+\frac{60}{h^2}e_4-\frac{120}{h^3}e_5\right)S\left(e_1+e_2-\frac{12}{h}e_3+\frac{60}{h^2}e_4-\frac{120}{h^3}e_5\right)^T \\
&- 2\left(e_1-\frac{1}{h}e_3\right)Z\left(e_1-\frac{1}{h}e_3\right)^T \\
&- 16\left(\frac{-1}{2}e_1-\frac{1}{h}e_3+\frac{3}{h^2}e_4\right)Z\left(\frac{-1}{2}e_1-\frac{1}{h}e_3+\frac{3}{h^2}e_4\right)^T \\
e_0 &= Ae_1 + A_1e_2 + A_2e_3
\end{aligned}$$

and $e_i \in R^{5n\times n}$ are elementary matrices, for example $e_2^T = [0 \quad I \quad 0 \quad 0 \quad 0]$.
Proof: Consider a Lyapunov-Krasvoskii canditate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$$

where

$$V_1(t) = \eta^T(t)P\eta(t), \quad V_2(t) = \int_{t-h}^{t} x^T(\alpha)Qx(\alpha)d\alpha$$

$$V_3(t) = h\int_{t-h}^{t}\int_{\beta}^{t} \dot{x}(\alpha)^T S\dot{x}(\alpha)d\alpha d\beta \quad \text{and} \quad V_4(t) = \int_{t-h}^{t}\int_{\beta}^{t}\int_{\sigma}^{t} \dot{x}(\alpha)^T Z\dot{x}(\alpha)d\alpha d\sigma d\beta.$$

where

$$\eta(t) = col\left[x(t), \int_{t-h}^{t} x(\alpha)d\alpha, \int_{t-h}^{t}\int_{\beta}^{t} x(\alpha)d\alpha d\beta, \int_{t-h}^{t}\int_{\beta}^{t}\int_{\sigma}^{t} x(\alpha)d\alpha d\sigma d\beta\right]$$

The time derivative $V(t)$ along the trajectories of system can be computed as follows:

$$\begin{aligned}
\dot{V}_1(t) &= 2\eta^T(t)P\dot{\eta}(t) = 2\xi^T(t)\Gamma P\Upsilon^T\xi(t) \\
\dot{V}_2(t) &= x^T(t)Qx(t) - x^T(t-h)Qx(t-h) \\
\dot{V}_3(t) &= h^2\dot{x}^T(t)S\dot{x}(t) - h\int_{t-h}^{t}\dot{x}(\alpha)^T S\dot{x}(\alpha)d\alpha \\
\dot{V}_4(t) &= \frac{h^2}{2}\dot{x}^T(t)Z\dot{x}(t) - \int_{t-h}^{t}\int_{\beta}^{t}\dot{x}(\alpha)^T Z\dot{x}(\alpha)d\alpha d\beta
\end{aligned}$$

where

$$\xi(t) = col\left[x(t), x(t-h), \int_{t-h}^{t} x(\alpha)d\alpha, \int_{t-h}^{t}\int_{\beta}^{t} x(\alpha)d\alpha d\beta, \int_{t-h}^{t}\int_{\beta}^{t}\int_{\sigma}^{t} x(\alpha)d\alpha d\sigma d\beta\right]$$

and it can be rewritten as

$$\dot{V}(t) = \xi^T(t)\left\{\Gamma P \Upsilon^T + \Upsilon P \Gamma^T + e_1 Q e_1^T - e_2 Q e_2^T + h^2 e_0 S e_0^T + \frac{h^2}{2} e_0 Z e_0^T\right\}\xi(t)$$
$$- h\int_{t-h}^{t} \dot{x}(\alpha)^T S \dot{x}(\alpha) d\alpha - \int_{t-h}^{t}\int_{\beta}^{t} \dot{x}(\alpha)^T Z \dot{x}(\alpha) d\alpha d\beta.$$

Applying Lemma 2.1 and 2.4 to the above integrals leads to

$$\begin{aligned}-h\int_{t-h}^{t} \dot{x}(\alpha)^T S \dot{x}(\alpha) d\alpha \leq -\xi^T(t)\Bigg\{ & (e_1 - e_2)\, S\, (e_1 - e_2)^T \\ & + 3\left(e_1 + e_2 - \frac{2}{h}e_3\right) S \left(e_1 + e_2 - \frac{2}{h}e_3\right)^T \\ & + 5\left(e_1 - e_2 + \frac{6}{h}e_3 - \frac{12}{h^2}e_4\right) S \left(e_1 - e_2 + \frac{6}{h}e_3 - \frac{12}{h^2}e_4\right)^T \\ & + 7\left(e_1 + e_2 - \frac{12}{h}e_3 + \frac{60}{h^2}e_4 - \frac{120}{h^3}e_5\right) \\ & S\left(e_1 + e_2 - \frac{12}{h}e_3 + \frac{60}{h^2}e_4 - \frac{120}{h^3}e_5\right)^T \Bigg\}\xi(t)\end{aligned}$$

$$\begin{aligned}-\int_{t-h}^{t}\int_{\beta}^{t} \dot{x}(\alpha)^T Z \dot{x}(\alpha) d\alpha d\beta \leq -\xi^T(t)\Bigg\{ & 2\left(e_1 - \frac{1}{h}e_3\right) Z \left(e_1 - \frac{1}{h}e_3\right)^T \\ & + 16\left(\frac{-1}{2}e_1 - \frac{1}{h}e_3 + \frac{3}{h^2}e_4\right) Z \left(\frac{-1}{2}e_1 - \frac{1}{h}e_3 + \frac{3}{h^2}e_4\right)^T \Bigg\}\xi(t)\end{aligned}$$

Hence, we have

$$\begin{aligned}\dot{V}(t) &\leq \xi^T(t)\left\{\Gamma P \Upsilon^T + \Upsilon P \Gamma^T + \Psi\right\}\xi(t) \\ \dot{V}(t) &\leq \xi^T(t)\Xi\xi(t).\end{aligned}$$

This completes the proof.

4. NUMERICAL EXAMPLES

In this section, two examples are used to illustrate the effectiveness of the proposed method.

4.1. Example:

Consider the following system with distributed delay:

$$\dot{x}(t) = \begin{bmatrix} 0.2 & 0 \\ 0.2 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} x(t-h) + \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} \int_{t-h}^{t} x(s) ds$$

The purpose is to match the utmost allowable upper bounds of h that guarantees the asymptotic stability of the above system. Table 4.1 lists the computed maximum allowable upper bounds and also the number of decision variables which keep the system stability by different methods. From Table 4.1, it's clear that the proposed approaches can provide higher upper bounds than those within the existing results. It should be noted that our method provides maximum allowable boundary which is adequate to the analytical bound with fewer number of decision variables.

Table 4.1. Upper bounds on h obtained for Example 4.1

Methods	Maximum h allowed	NoDv
Chen and Zheng 2007	1.6339	85
Seuret and Gouaisbaut 2013	1.877	16
Park et al. 2015	1.9504	59
Zeng et al. 2015	2.0395	75
Trinch 2015	2.0395	27
Zhao et al. 2017	2.0402	45
Park et al. 2018	2.0412	42
3.1 Theorem	2.0412	39
Analytical Bound	2.0412	-

4.2. Example:

Consider the following system with distributed delay:

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} x(t-h) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \int_{t-h}^{t} x(s)ds$$

Table 4.2 lists the computed upper bounds by different methods and it shows that our method provides an upper bound which is quite close to the analytical bound.

Table 4.2. Upper bounds on h obtained for Example 4.2

Methods	Maximum h allowed	NoDv
Zhao et al. 2017	6.1663	45
Zeng et al. 2015	6.1664	75
Gu et al. 2003 (N=3)	6.171	67
Chen et al. 2016	6.1719	106
Park et al. 2018	6.1719	42
3.1 Theorem	6.1719	39
Analytical Bound	6.1725	-

5. CONCLUSION

In this article, delay dependent stability criteria for linear time-delay system with distributed delay are proposed by the employment of the Lyapunov method . By the development of augmented Lyapunov functional and relaxed double integral inequality, the delay dependent stability criterion has been proposed. Compared to the recently proposed integral inequalities the obtained ones could provide more accurate estimations on the handling of the integral terms arising within the derivative of the Lyapunov-Krasovskii functionals(LKFs) The obtained stability condition provides larger feasible solution region and fewer conservatism with fewer number of decision variables than the present ones within the literature. Two numerical examples are presented to indicate the effectiveness of the proposed approach.

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