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Evolutionary Game to Model Risk Appetite of Individual Investors

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Abstract: In this paper, we present a novel mathematical model of the development and progression of investments based on evolutionary game theory. We consider four different types of investments in the market: bank account, bond, stocks, and risky derivatives. Despite the relative sincerity of the model, it supplies a way to explore the interactions between the different investment types in the market. We assume the market is complete and four assets constitute the total capital market. We also assume that the risk-averse individual, the risk-neutral individual, and the risk-seeking individual enter the market, and that each of these three individuals buys at least two assets. We examine the interaction of the two assets and find evolutionary stable strategy in interactions and weights. We explore the heard effect on decision-making and investment. By providing examples in the capital market, we find that the results are consistent with the conditions in the capital market. The results show that investors who enter the market may change their behavior due to their herds effect and tend to other strategies that are more appropriate according to evolutionary game theory.

Keywords: game theory, portfolio risk, expected payoff, evolutionary stable strategy, heard effect

1. INTRODUCTION

In this paper, we examine the application of evolutionary game theory the risk and present a new mathematical model of investment development based on evolutionary game theory. A branch of mathematics is game theory such that analyzes interactions involving strategic financial decision-making. This interaction is called a game. The parties included in the game are the players, who can be persons or groups. The players are assumed to act rationally [1]. Game theory has controlled many areas, from economics (historically its initial focus) to political science to biologies, such as business, wireless networks, computer science, psychology, and many others [2, 3].
Evolutionary games examine behaviors in evolving populations. The goal is to examine changes in population over time. In human interactions, players who gain a good experience of their choice in one period pass it on to those around them so that they can make the same choice in the next round. Humans interact with each other in many situations, and many of their situations and interactions depend on financial circumstances. In these interactions, it is important to choose the best strategy that will enable people to achieve the highest results. Analysis and understanding of situations in which decision-makers are interacting is one of the goals of game theoretic.

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Risk attitude is a concept of decision theory and allows individuals to determine their preferences from identified risks. Game theory helps solve risk problems and risk management. Game theory is combined with risk management theory to capture any opportunity that a business can turn the risk into opportunity. Game theory for managers dealing with a huge amount of data can be an alternative perspective that they can view the process of solving hard problems. Some studies have shown how game theory helps managers to solve problems, manage risks and make strategic decisions. Financial risk is the possibility of losing money on an investment or business activity and is a type of risk that can lead to the loss of capital for investors. Risk analysis supports game theory by providing various possibilities. Methods and concepts of game theory can be effective in improving risk analysis. Also, risk analysis and game theory are complementary [4].
Investors have different and complex behaviors due to interaction with each other. If we have a solution for their behavior when entering the market and their behavior is predictable, then other people can choose the best strategy when entering the market and investing and have a more successful investment. Given that investors have different preferences, in this article, we are looking for a solution to be able to predict the behavior of these investors using evolutionary game theory in the market and express the best strategy for them. In this article, we answer the following questions in the capital market:

- Which strategy investors choose based on their behavior in the capital market?
- In the capital market, which strategy for investors is an evolutionary stable strategy?
- Why do investors tend to pursue this evolutionary stable strategy?
- Is it possible to predict the behavior of investors in terms of their type in the capital market?

There are models for people's risk appetite using game theory. The model presented in this article for investors' risk appetite is a complete, logical, and close to a reality model. Because if we compare this model with the Nash model, we will see that each investor has chosen his strategy based on his opinion, but in the proposed model, each investor inherently has his own strategy and we know that in the real world people are inherently risk-seeking, neutral-risk, and risk-averse, so this model makes more sense. Also, in Nash model, players represent real players, but in our model, players (investors) represent players who are randomly selected from the population, and because the risk appetite of investors in a population is examined, so our model which expresses the choice of investors in the population is a model close to reality. Therefore, our model shows this correct choice. Nash model does not pay much attention to the mechanism of change over time, but our model examines changes in representative population strategy over time. Also, in Nash model, equilibrium is obtained, but in the proposed model, we obtain ess, which is the more general state of Nash equilibrium. Comparing this model with the Nash model, we found that this model is an efficient model to examine the risk appetite of investors and provide the best strategy in the population of investors, and this model can be used to predict the behavior of investors and express their optimal strategy.

2. LITERATURE REVIEW

Evolutionary game theory is one of the important theories. Numerous studies and researches in the field of evolutionary games have been done by scientists and researchers. We will review some of the most important of these researches. Evolutionary Game Theory began with an article by Price and Maynard Smith [5]. Evolutionary Game Theory (in short EGT) has burgeoned into a base of computational biology and mathematical [6–8]. The mathematical foundations of game theory experienced a revival when Price and Maynard Smith revolved their attention to Evolutionary Game Theory. Maynard Smith coined the term Evolutionary Stable Strategy (in short ESS), to mean a move (or play) that would assure the type of animal that wields it an evolutionary advantage over their rival, with the meaning that the Evolutionary Stable Strategy can not ever be extinct [9].

Using the basic tools in hand, continuous-trait game theory can be clearly extended to model evolution under situations of disruptive choice and nonequilibrium population dynamics, stochastic environments, speciation, coevolution, and others. Multitude models applying these tools to evolutionary ecology and coevolution have been developed in the past two decades. Trait evolutionary game theory and new applications are important in biological issues [10].

In the basic description of altruistic behaviors in Darwinian evolution, evolutionary game theory has helped. It has in turn become of interest to sociologists, anthropologists, philosophers, and economists. Evolutionary game theory is used in many areas, including sociology, economic, anthropology, social networks political science, and social sciences [11, 12]. In 1990 "An Evolutionary Game Theory Model for Risk-Taking" was published by S Ellner and A Shmida S Ellner and A Shmida in 1990 in a paper entitled "An Evolutionary Game Theory Model for Risk-Taking" published [13]. In 1998, the financial applications of game theory were proposed by Franklin Allen and Stephen Morris. In this study, it is stated that traditional financial theory based on the assumptions of symmetric information and complete and competitive markets has provided many insights [14]. Using theory of game, risk analysis can be based on values of benefit or preferences which the objects can supply rather than subjective probability. Furthermore, it can also be used in settings where statistical data is unavailable. This may expand the appropriateness and quality of the overall risk analysis process.

An article was written by James Raneke in 2009 on a game theory formula that discusses decision-making in terms of uncertainty and risk. The game environment is examined to decide in terms of uncertainty and risk for a general set of issues. The method is demonstrated using the assumptions governing future oil prices and environmental degradation to evaluate long-term investment options [15].

In 2011, Rajabandari and Snekkenes published an article entitled "Using Game Theory for Private Risk Analysis: A Basic Insight". In this paper, the authors state that with the advancement of information technology, there is a growing risk of privacy due to the widespread use of identity information. This article discusses some of the key issues surrounding the use of game theory in privacy risk analysis. Using game theory, risk analysis can be based on preferences or profit margins [16]. The results and reasoning of game theory have overshadowed economic theory widely used in other behavioral and social sciences. Also, it is widely used in economic decisions and other gaming situations that involve risk coalition game theory for security risk management. This paper examines the possibility of cooperation between the autonomous departments of an organization with dependent security assets or vulnerabilities to reduce overall security risks. A coalition game has been formed to model the possibilities of cooperation in these sectors based on their positive and negative interdependencies. The proposed game forms a framework that allows us to examine how an organization can maximize its overall profitability through collaboration between its various departments [17].

In 2012, Farshchi wrote an article about finding the dominant strategy for risk in incomplete play discussed. He argued that game theory reasoning exists in economic theory and is widely used in other social and behavioral sciences, and that it is widely used in decision-making about economic behavior and other risk-playing conditions. The paper proposes a flawed multiplayer information system using a dominant risk strategy for decision-making. Finally, it discusses the operation of the strategy [18].

In 2013, an article on financial markets and evolutionary beliefs was published by Ellis Jovini, Clotilde Knop, and Yannick Wyt [19]. 2016 Financial risk of supply chain of small and medium enterprises based on evolutionary game theory by Cheng Zhang Checked out. This article examines the game between commercial banks and corporate supply chains. And examines the stability of the balance between it Zhang.

In 2018 "An Evolutionary Game Model for Behavioral Gambit of Loyalists: Global

Awareness and Risk-Aversion" published by E Alfinito, A Barra, M Beccaria, and A Fachechi [20]. In 2018, Ashkan Hafezalkotob published an article entitled "Market Risk and Uncertain Demand in the Supply Chain". The paper argues that uncertainty in market demand poses a significant risk to retailers who supply the market, and that retailers' risk-aversion behavior toward risk may have changed over time. Considering a supply chain including the manufacturer and the retail population, the authors intend to examine how the retail population evolves toward risky behaviors. The purpose of this study is to evaluate the effects of producer wholesale selling price on retailers' sustainable evolutionary strategy. A supply chain with a population of risk-averse and neutral risk retailers is examined. The wholesale pricing strategy is determined by a manufacturer who acts as a leader, while the retailers who make quantitative decisions act as the leader. To model this situation, Kornot dual equilibrium approach and evolutionary game theory have been used [21]. In 2018, "Evolutionary Game Dynamics for Financial Risk" was studied. In the same year, Zhi Li, Guanghao Jin, and Shen Duan conducted a study on global supply chain performance based on EGT. They wrote the evolutionary game between the main player in the supply chain and, using replicator dynamics, developed an ESS for the game. The authors used evolutionary game theory to model the performance of supply chain members in a financial risk environment. The results of this study showed that in high financial risk, the cooperation strategy is a sustainable evolutionary strategy for both producers and suppliers [22].

Game theory has a lot of applications in the business world. In 2019, a book entitled "Introduction to Derivative Securities, Financial Markets and Risk Management" was written by Jarrow and Chatterjea. This book explores the interest rates on derivatives, stocks, options, trade, financial engineering, futures markets, futures, and other issues [23].

In 2020, an article entitled "Heterogeneity in Evolutionary Games: An Analysis of Risk Perception" was written by Marco A. Amaral and Marco A. Javarone. Which examines the relationship between cooperation and heterogeneity. The approach of this modeling is based on the framework of evolutionary game theory [24]. "The Evolutionary Game of Stakeholders ' Coordination Mechanism of New Energy Power Construction PPP Project: A China case" was released by L Gao and ZY Zhao in 2020. In this study, government, investors, and the general public have established a relationship with multiple dynamic public-private partnership (PPP) play. The authors first developed a three-dimensional game model consisting of government, investors, and the people based on evolutionary game theory. In the second stage, they study the three-way evolution of strategic behaviors with a system dynamics model. Finally, the effect of changes in key factors in behavioral strategies was studied and the results showed that the three parties eventually reach equilibrium and the benefits of the project are at their highest at this time and the three parties achieve a win-win situation. Also, the security factor plays a vital role in choosing the overall strategy. Public choice is affected not only by its own income and expenditure, but also by the amount of compensation promised by the government and the amount estimated by the investor [25].

In 2021, an evolutionary game theory model for the relationship between financial regulation and financial innovation was proposed by Hui An, R Yang, X Ma, and S Zhang. This paper builds an evolutionary game model for simulating financial regulation and financial innovation to analyze sustainable equilibrium strategies between financial institutions and regulators. The article also uses US macroeconomic financial data [26].

3. EVOLUTIONARY GAME THEORY (EGT)

EGT be created as an application of the mathematical theory of games to biological content, arising from the awareness that frequency-dependent fitness establishes a strategic aspect to evolution.

The attention among scientists and scholars in theory with detailed biological roots derives from three facts. First, the evolution process by evolutionary game theory require not be biological evolution. Evolution can, in this situation, often be understood as cultural evolution, where this refers to changes in norms and beliefs over time. Second, the rationality suppositions underlying evolutionary game are in lots categories more appropriate for the modelling of social systems than those suppositions underlying the classical game theory. Third, EGT, as a clearly dynamic theory, supplies an important factor missing from the classical theory.

EGT has been used to describe a large number of aspects of human behavior. Evolutionary games have important unrealized potential for modeling substantive economic issues [11]. Evolutionary game theory is used in different sciences, such as social dilemmas, economic, sociology, and anthropology [27,28].

The beginning of evolutionary game theory can be dated to the definition of an evolutionarily stable strategy (ESS) by Price and Maynard Smith. In certain populations, all individuals be possible have the same strategy phenotype. Such a strategy is called to be an ESS if that strategy cannot be replaced, or invaded by any other strategy through natural selection. In the EGT, the concept of evolutionary stable strategy is a population in which all those who play this strategy are resistant to the invasion of a group of mutant persistent.

Function u represents a player 's payoff. In short, the following definition is given:

α is an (ESS) such that:

For each $\beta \neq \alpha$, there are some $\varepsilon' \in (0,1)$, which may depend on β, such that: for all $\varepsilon \in (0,\varepsilon')$

$$u(\alpha, \varepsilon\beta + (1-\varepsilon)\alpha) > u(\beta, \varepsilon\beta + (1-\varepsilon)\alpha). \tag{3.1}$$

That is, α is (ESS) if, after mutation, non-mutants are more successful than mutants, in which exposition mutants cannot invade and will eventually get extinct.

4. PORTFOLIO OPTIMIZATION MARKOWITS APPRCACH

In financial affairs, Markowitz model-put forward by H.Markowitz is a portfolio optimization model, this model helps in the option of the most efficient portfolio by studding various possible portfolios of the given securities. Markowitz made the following hypothesizes:

Risk of a portfolio is established on the variability of returns from the said portfolio. An investor is risk-averse. An investor would like to increase consumption. The investor's utility function is increasing and concave, due to his risk aversion and consumption preference. Examination is based on the alone period model of investment. An investor either maximizes his return for the minimum risk or maximizes his portfolio return for a given level of risk. An investor is rational in nature.

A portfolio that gives maximum return for a given risk, or minimum risk for given return is an efficient portfolio. Therefore, portfolios are Thus, portfolios are chosen as follows:

An investor will prefer from the portfolios that have the same return, the portfolio with lower risk. And the investor will prefer the portfolios that have the same risk level from the portfolio with a higher rate of return. Because he is a reasonable investor, he tends to be more profitable, and because he is a risk averse, he tends to take less risk.

For choice of the best portfolio, the risk-return preferences are studied. An investor who is highly risk-averse will hold a portfolio on the lower left hand of the frontier, and an investor who is not too risk-averse will select a portfolio on the upper portion of the frontier [29,30].

5. PROSPECT THEORY

First Allais, researchers noticed that expected utility theory (in short EUT) doesn't sufficiently explain decision-making under risk.

The leading alternative to expected utility theory, prospect theory, supposing that individuals appraise gains differently from losses:a gain profit persons less than a loss of the same size

benefits them.
On the other hand, the weights that a person attributes to outcomes of risky decisions do not coincide with the probabilities of the respective outcomes. This probability weighting typically leads to over weighting of low probabilities and under weighting of high probabilities.
One of economics theory is the prospect theory that improved by Amos Tversky and Daniel Kahneman. It challenges EUT, improved by Oskar Morgenstern and von Neumann.
It is the founding theory of behavioral finance and of behavioral economics, and comprises one of the first economic theories created using experimental manners. Founded on results from regulated studies, it explains how persons appraise in an asymmetric mode their gain and loss perspectives.
For example, for some persons, the hurt from losing $\$500$ could just be paid back by the pleasure of $\$1,000$. The prospect theory begins with the idea of loss aversion, an asymmetric form of risk aversion, from the observation that people react differently between potential losses and potential gains. Consequently, people make decisions founded on the potential losses or gain relative to their specific condition rather than in absolute terms: Facing a risky choice leads to profit, persons are risk-averse, appointing solving that lead to a lower expected utility but with a higher definiteness (function is concave). Facing a risky choice leading to losses, persons are risk-seeking, appointing solutions that lead to a lower expected utility as long as it has the ability to nullify losses (function is convex). These examples are so in contrast with EUT, which just considers options with the maximum utility. EUT has influenced the analysis of decision-making under risk.
Generally, it is accepted as a normative model of rational choice, and greatly applied as an illustrative model of economic behavior [31–33].
Consequently, it is supposed that all credible people would wish to obey the axioms of the theory, and that most people in factf do, most of the time [34, 35].
The prospect theory probability value function $v(r)$ and weight function $\pi(p)$ [36].
$v(r)$ explicit formula is the value function of the prospect theory that is as follows [37]:

$$v(r) = \begin{cases} (r - r_0)^{\alpha} & \textit{if} \quad r \geq r_0 \\ -\lambda(r_0 - r)^{\beta} & \textit{if} \quad r < r_0 \end{cases} \tag{5.2}$$

Where (r_i) mean return of asset i, (r_0) reference point,(λ) is positive value of risk and $(-\lambda)$ is negative value of risk, α and β are risk aversion coeffcients with respect to gains and losses respectively.
The prospect theory utility function can be written in terms of v and π as:

$$PT_U = \sum_{s=1}^{S} \pi(P_s)v(r_s) = \sum_{s=1}^{S} P_s v \left(\sum_{i=1}^{N} r_{si} w_i \right). \tag{5.3}$$

The parameters are as follows:
(P_s) the probability of scenario s,
(r_{si}) the return of asset i in scenario s, $i = 1, \ldots, N$, $s = 1, \ldots, S$,
(w_i) the weight of asset i in the portfolio, $w_i \geq 0$,
(S) the number of scenarios,
(r_i) mean return of asset i,
(r_0) reference point,
(N) the number of assets.
The diagram prospect theory is shown in Figure 5.1 [37].

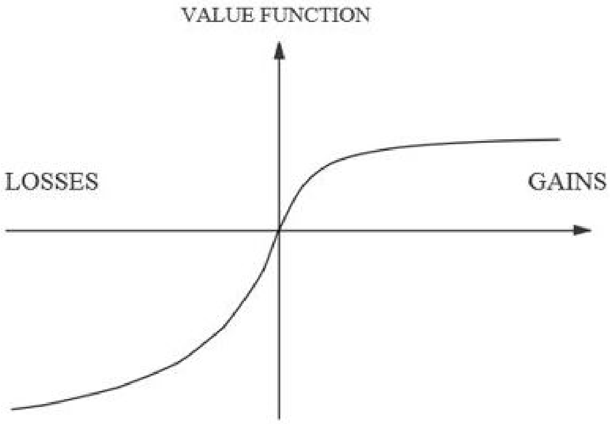

Fig. 5.1. Prospect theory value function $v(r)$ with $\lambda = 2.25$ and $\alpha = \beta = 0.88$.

6. GAME THEORY AND PROSPECT THEORY

Extant research includes lots of effective applications of prospect theory, specially in insurance, finance, and betting markets.
However, prospect theory studies are similar in that the risk that people face is exogenous.
In lots of real-world conditions, contradiction, risk does not come from an exogenously given random device but rather is a result of the interaction of one person with others.
In developing the prospect theory model, show (I) a solution strategy for incorporating prospect theory into a dynamic game theoretic model and its empirical applicability to field data, (II) the usefulness of prospect theory in depicting people behavior, and (III) managerial connotations that result from using prospect theory over the more commonly used expected utility theory.
When we enter a game and play that game, payoffs are mostly given in monetary (and other similar) amounts. As Morgenstern and von Neumann observed, game theory has therefore to take into account the players preferences on uncertain and sure monetary outcomes of game. To incorporate prospect theory into a game theoretical framework, at first glance looks straightforward: the naive approach would be to transform the monetary outcomes via the value function and to transform actual probabilities for chance moves into experienced probabilities by implementing the probability weighting functions. This procedure mimics the technique one successfully applied when dealing with EUT.
Focusing on the theory of perspective, which is devoted to human behavior in risky financial decisions, the idea of the expected utility theory has been developed by adding psychological components that consider human behavior in the decision-making process. If we want to show the minimum risk in choosing a stock portfolio with a fixed return, with a behavioral approach versus the traditional Markowitz approach, we can impose a behavioral approach on the traditional approach. The reason for imposing behavioral characteristics on the theory of perspective is to minimize the risk with a certain level of income, in the matter of choosing a portfolio. According to the prospect theory portfolio selection problem looks as follows:

$$\textit{Maximize } PT_x = \sum_{s=1}^{S} P_s v \left(\sum_{i=1}^{N} r_{si} w_i \right). \tag{6.4}$$

Where (P_s) the probability of scenario s, (w_i) represents the weight of asset i in the portfolio, (r_{si}) the return of asset i in scenario s.

Subject to the constraints

$$\overline{r}(x) = \sum_{i=1}^{N} \overline{r}_i w_i \geq d, \tag{6.5}$$

$$\sum_{i=1}^{N} w_i = 1. \tag{6.6}$$

$$w_i \geq 0, \quad i = 1, \ldots, N$$

Where (w_i) represents the weight of asset i in the portfolio, $w_i \geq 0$, (d) represents the desirable level of return, (r_i) mean return of asset i.

7. NEW MODEL DEVELOPMENT

In this chapter, we examine the issue of people's risk appetite for investment and present this issue with the evolutionary game theory approach. Also, we develop a new model to investigate the investment strategies in real market. Our model includes a demonstration of real-world investment. Investors have different and complex behaviors due to interactions with each other. This diversity and complexity of investor behavior leads to a model in evolutionary game theory. This mathematical model is used to examine interactions between investors. We assume that three types person enter the market. Risk appetite reflects the willingness of investors to take risks. We express the preferences of investors using a level of risk that is acceptable to them. We denote the person whose behavioral preferences are risk-averse by p_1, person whose behavioral preferences are risk- neutral by p_2, and person whose behavioral preferences are risk-seeking by p_3.
The risk-averse agent prefers to play the strategy that takes the least risk. A risk-neutral agent only attempts to get the highest expected utility. A risk-seeking agent chooses to play the strategy that gives the highest payoff without taking the risk into account.
Suppose the x-axis represents income and the y-axis represents utility. The graph of the utility function of these individuals is shown in Figure 7.2.

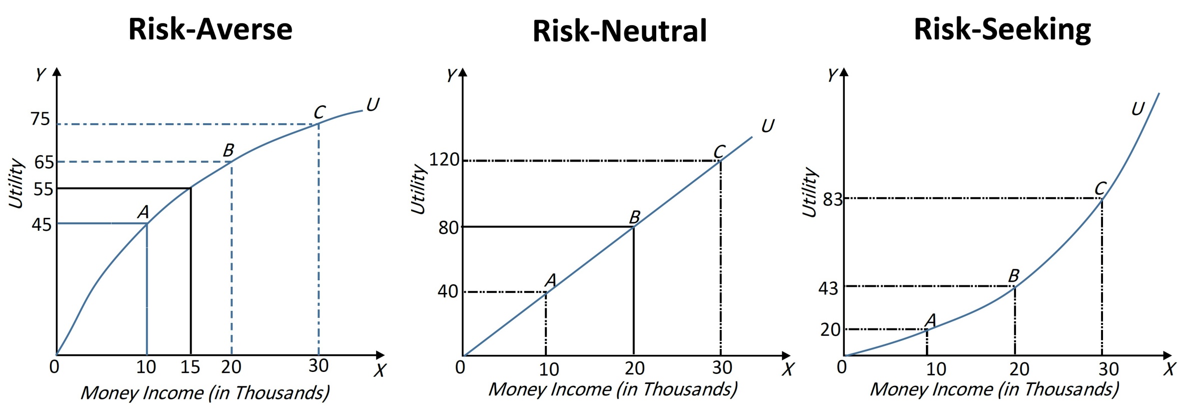

Fig. 7.2. The utility of individuals who enter the market.

We assume there are four different investment types in the market which the players may invest with respect of risk and return of each asset and his own preferences. These are four assets which are: investment on a bank account, bond, stock, and risky derivatives.

We assume the market is a complete market and we assume that the assets on which the bank account, bond, stocks, and derivatives constitute the total capital market. The complete market occurs when all the assets are invested, and their behavior affects others.
We consider that there are four types of assets in the market:

(a): Bank account
(b): Bond
(c): Stocks
(d): Risky derivatives

The weight of the various types of assets are denoted by A_a, A_b, A_c, and A_d respectively. The weights are satisfy

$$\sum_{i=a}^{d} A_i = \text{unity}, \tag{7.7}$$

$$0 \leq A_i \leq 1.$$

The following relationship applies to all assets.

$$\sum (A_i \times W_i) = \sum C_i. \tag{7.8}$$

Where C is the total capital of the individuals and can be defined.
We will now set up costs and benefits for each type of asset.
The notation and model parameters are summarized in Table 7.1.

Table 7.1. Model parameters

symbol	meaning
A_a	weight of the bank account
A_b	weight of the bond
A_c	weight of the stocks
A_d	weight of derivative
μ	benefits of asset i
r	risk of asset i
CP	the cost of call price
DP	the cost of discounted price
Eps	the earning per share
W_0	initial fitness
W_X	net benefit for a given type of the investment $X \in \{a, b, c, d\}$

The concept of risk plays a key role in the financial market. Risk is the main pillar of decision-making for investment. Investors who start investing regardless of risk will make themselves in a loss-making situation. We consider this risk and show with r. Earning per share (Eps) is the earning obtained by each share stock.
Call price is the value at which a corporation can purchase and exercise preferred stock from its callable preferred derivating which shows the cost of call price with CP. Discount price is reduced prices or something being sold at a price lower than that item is normally sold for which shows with DP.
Asset a indicates the bank accounts which is the symptom of the minimax risky asset. Asset b indicates the bonds with the lowest investment risk and asset c is the stock with market fluctuations. Asset d is all the other types of risky derivatives which indicates the highest risk related investment in the market.
we have:

$$r_a < r_b < r_c < r_d, \tag{7.9}$$

And

$$\mu_a < \mu_b < \mu_c < \mu_d. \tag{7.10}$$

When the weight of the investment A_a, A_b, A_c and A_d, using the symbols in Table 7.1, the net benefits to each type of the investment are

$$\begin{aligned} W_{A_a} &= W_0 - DP + \mu(A_a + A_b) - r(A_a + A_b), \\ W_{A_b} &= W_0 - \text{Eps} + \mu(A_a + A_b + A_c) - r(A_a + A_b + A_c), \\ W_{A_c} &= W_0 - \text{Eps} + \mu(A_b + A_c + A_d) - r(A_b + A_c + A_d), \\ W_{A_d} &= W_0 - CP + \mu(A_c + A_d) - r(A_c + A_d). \end{aligned} \tag{7.11}$$

W_{A_a} reads the individual in the investment bank has the initial fitness W_0, pays the cost of discounted pric DP, receive benefits from investment in a and b, shown by the term $\mu(A_a + A_b)$ and pays the cost of risk, shown by the term $r(A_a + A_b)$.
W_{A_b} reads the individual in the investment bond has the initial fitness W_0, pays earning per share Eps, receive benefits from investment in a, b, and c, shown by the term $\mu(A_a + A_b + A_c)$ and pays the cost of risk, shown by the term $r(A_a + A_b + A_c)$.
W_{A_c} reads the individual in the investment stocks has the initial fitness W_0, pays earning per share Eps, receive benefits from investment in b, c, and d, shown by the term $\mu(A_b + A_c + A_d)$ and pays the cost of risk, shown by the term $r(A_b + A_c + A_d)$.
W_{A_d} reads the individual in the investment stocks has the initial fitness W_0, pays the cost of call price CP, receive benefits from investment in c and d, shown by the term $\mu(A_c + A_d)$ and pays the cost of risk, shown by the term $r(A_c + A_d)$.
The payoff matrix between the different assets is shown in Table 7.2 .

Table 7.2. The payoff matrix

	a	b	c	d
a	$\mu - r$	$\mu(a+b) - r(a+b) - DP$	0	0
b	$\mu(a+b) - r(a+b) - DP$	$\mu - r$	$\mu(b+c) - r(b+c)$ $-DP - \text{Eps}$	0
c	0	$\mu(b+c) - r(b+c)$ $-DP - \text{Eps}$	$\mu - r$	$\mu(c+d) - r(c+d)$ $-CP - \text{Eps}$
d	0	0	$\mu(c+d) - r(c+d)$ $-CP - \text{Eps}$	$\mu - r$

For example, in Table 7.2, the only payoff obtained is $\mu - r$ when the asset a is interacting with another asset a, when an asset a is interacting with the asset b, the payoff is $\mu(a + b) - r(a + b) - DP$, when the asset a is interacting with another asset c The payoff is 0 because they are not interacting with each other, and similarly, when an asset a is interacting with the asset d, the payoff is 0.

8. EMPIRICAL APPLICATION

Assuming that the investors were three types: risk-averse individual, risk- neutral individual, and risk-seeking individual. We assume that one kind of individual enters the market from these three types of individuals.
And we assume that individuals purchase at least two assets. Assuming a risk-averse individual enters the market. This person only evaluates a and b. This person is not interested in investing in c and d because the person is risk-averse.
So, in this case, the bank account and the bond are interacting. In situations where bank account and bond are interacting, we are interested to know which is an evolutionary stable strategy. Meaning that we find evolutionary stable strategy in this interaction.
But the condition ESS of such a situation is:

$$W_{A_a} - W_{A_b} = \text{Eps} - DP - \mu A_c + rA_c. \tag{8.12}$$

Recall that the net benefit from interaction (fitness) for the asset bank account is denoted by W_{A_a} and for the bond, by W_{A_b}. We have:

$$W_{A_a} = W_{A_b}. \tag{8.13}$$

Then

$$A_c = \frac{\text{Eps} - DP}{\mu - r}. \tag{8.14}$$

As the risk-averse enters the market, equilibrium is c and investor prefers to choose c. This result is due to the herd effect. That is, there is a herd effect in the financial market when a group of investors ignores their own information and instead only follows the decisions of other investors.

So, in this situation, if every other risk-averse individual enters into the market, it will bring the same choice c, and therefore the whole population will choose this strategy. And this strategy is an ESS.

Assuming a risk-neutral individual enters the market. This person only evaluates b and c. And is not interested in investing in a and d because the person is risk-averse. Because assets a has low income and assets b are high risk, the person is unwilling to choose them.

So, in this case, the bond and the stocks are interacting.

We will now find the conditions evolutionary stable strategy in interaction between bond and stocks.

But the condition ESS of such a situation is:

$$W_{A_b} - W_{A_c} = \mu A_a - rA_a - \mu A_d - rA_d, \tag{8.15}$$

$$A_a = \frac{\mu A_d - rA_d}{\mu - r}. \tag{8.16}$$

Or

$$A_d = \frac{\mu A_a - rA_a}{\mu - r}. \tag{8.17}$$

As the risk-neutral individual enters the market equilibrium are a or d and investor prefers to choose a or d. In this case, the investor behavior changes due to the herd effect. That's mean, when a risk- neutral enters the market, it is influenced by the behavior of others, the best strategy for this individual is the choice of a or d, So chooses a or d.

Then a and d are mixed evolutionary stable strategy.

In the evolutionary game theory a situation where two or more types are best suited to one of the other types, mixed evolutionary stable strategy is called.

Assuming a risk-seeking individual enters the market. This person only evaluates c and d. Because the person is risk-seeking, this person is looking for higher incomes then is not interested in a and b.

So, in this case, the stocks and the risky derivatives are interacting.

Now find the conditions evolutionary stable strategy in the interaction between stocks and derivatives.

But the condition ESS of such a situation is:

$$W_{A_c} - W_{A_d} = \mu A_b - rA_b + CP - \text{Eps}, \tag{8.18}$$

$$A_b = \frac{\text{Eps} - CP}{\mu - r}. \tag{8.19}$$

As the risk-seeking individual enters the market, equilibrium is b and investor prefers to choose b. That is, in this group of investors ignore their information and change their behavior and instead follow only the decisions of other investors, so there is a herd effect in the financial market.
Many times in financial markets, as in our daily life, we parrot the decisions of predecessors, instead of analyzing available information and making our own decisions. This following the decision can lead to collective hysteria and investment calls may be influenced by these panicked conditions.
Each investor gets information (or signals) about how they should act. They also know the decisions of their predecessors, although they don't know the signs they received. Using this knowledge, each investor makes their own decision. But this unawareness of previous information can sometimes cause investors to disregard the signals they get in the here and now, and instead take in the same decisions as their predecessors.
So, in summary a herd effect exists in the financial market when a group of investors ignores their own information and instead only follows the decisions of other investors.
Some studies (such as herd behavior in financial markets, or daring to be different) treat this effect as an intraday phenomenon, which intuitively makes sense. Investors are much more likely to parrot others than to explicate the information they get when they have a few times to make investment decisions.

9. ILLUSTRATION RESULTS

In this section, we review the research findings and compare the findings with the market results by providing an example. The data are derived from the European average 10-year interest rate on financial derivatives [38]. Each investor is interested in the risk and return of his investments in the market economy.
In this example, we consider four assets a, b, c, and d. Asset a is considered in bank accounts and we consider it as a risk-free investment. For asset a we consider the benefit to be 2, the risk of the asset to be 0.5 and the discount price (DPc) to be 4. Asset b is considered an investment on commodity. For asset b we consider the benefit to be 3.5, the risk of the asset to be 2 and discount rate to be (DR) to be 1.5. Asset c is considered investment on stocks and equity. For asset c we consider the benefit to be 4, the risk of the asset to be 34 and minimum Eps which is actually the lowest Eps in the stock market (min Eps d) to be 1.5. Asset d is considered investment on FOREX (curvency exchang) and we considered as a high risk investment. For asset d we consider the benefit to be 6, the risk of the asset to be 4.5 and cost of call price (CP) to be 0.5.
Consider Table 9.3 for the assets.

Table 9.3. Numerical examples

asset	benefit (%)	risk	weight	cost
a	$\mu_a = 2$	$r_a = 0.5$	A_a	$DP = 4$
b	$\mu_b = 3.5$	$r_b = 2$	A_b	$3.5 = 2 + DRc$
c	$\mu_c = 4$	$r_c = 3$	A_c	$3.5 = 2 + \text{min Eps } d$
d	$\mu_d = 6$	$r_d = 4.5$	A_d	$CP = 0.5$

We assume that a risk-neutral individual enters the market in this case, according herd effect, investor behavior changes and prefers to choose A_a because

$$A_a = \frac{\mu_d A_d - r_d A_d}{\mu_a - r_a} \to A_a = \frac{6(A_d) - 4.5(A_d)}{2 - 0.5}. \tag{9.20}$$

When a risk-neutral individual enters the market

$$A_d = \frac{\mu_a A_a - r_a A_a}{\mu_d - r_d} \to A_d = \frac{2(A_a) - 0.5(A_a)}{6 - 4.5}. \tag{9.21}$$

Then we have:

$$A_a = A_d. \tag{9.22}$$

In this case, this person has two choices A_a or A_d. that is, we have a mixed evolutionary stable strategy. It is a strategy in which two strategies are permanently present. Finally, these two strategies become pervasive in the population with the type of risk-neutral behavior.
On the other hand, when a risk-seeking individual enters the market according to herd effect, investor behavior changes and prefers to choose A_b and the whole population will go to this strategy and follow it because

$$A_b = \frac{\text{Eps} - CP}{\mu_b - r_b} \to A_b = \frac{3 - 0.5}{3.5 - 2}. \tag{9.23}$$

Finally, this strategy becomes pervasive in the population with the type of risk-seeking.
And when a risk-averse individual enters the market according to herd effect, investor behavior changes and prefers to choose A_b because

$$A_c = \frac{\text{Eps} - DP}{\mu_c - r_c} \to A_c = \frac{3 - 4}{4 - 3}. \tag{9.24}$$

Finally, this strategy becomes pervasive in the population with the type of risk-averse.
In this example, when three individuals enter the market with different preferences and buy from the four assets on the market, we have the following result in the weight of the assets:

$$\sum_{i=a}^{d} A_i = \text{unity}. \tag{9.25}$$

And in each interaction, investors tend to change their behavior because they are influenced by others experiences and prefer buying an asset that most people have bought. And as a result, this strategy is more appropriate than any other strategy, and the whole population will go to this strategy and follow it. This strategy is an ESS. The results presented in this numerical example correspond to real-world results (according to the herd effect) and therefore, this model is useful for investing and predicting investment results.

10. CONCLUSION

In this paper, we showed and analyzed a game-theoretical novel model of investments. We assumed the market is a complete market and the assets bank account, bond, stocks, and risky derivatives constitute the total capital market. Also, assumed that the risk-averse individual, risk-neutral individual and risk-seeking individual enter the market and each individual buys at least two assets. Using evolutionary game theory, we showed how each investor behaves according to their preferences when this investor enters to the market and which strategy is

an evolutionary stable strategy in the interaction between the two assets. We found that when risk-averse individual enters the market, two assets a and b are interacting, and A_c is an ESS. In this case, the investor behavior changes due to the herd effect and the whole population will go to this strategy. When risk-averse individual enters the market, it is influenced by the behavior of others and most people have invested in this asset.
When risk- neutral individual enters the market, two assets b and c are interacting, and A_a or A_d are mixed ESS. So, the investor behavior changes and most people have invested in these two assets. When risk-seeking individual enters the market, two assets c and d are interacting, and A_b is an ESS. In this case, the investor behavior has changed due to the herd effect and prefers to choose A_b and the whole population will go to A_b.
By giving an example, we find that the results are consistent with real-world results. Therefore, using the new model presented in this article, we can predict their choice when entering the capital market according to the type of behavior of investors and explain the reason for this choice according to the herd effect. Comparing the model presented in this article for investors' risk appetite and Nash model, we conclude that this model is a more complete, logical and close to a reality model.

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